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APPLYING GRAPH THEORY TO OPTIMIZE PRODUCT DELIVERY ROUTES AND MINIMIZE COSTS IN THE RESTAURANT BUSINESS

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ABSTRACT

An important role in the management system of restaurant business enterprises in conditions of competition belongs to solving complex tasks of managing product supply chains and developing new technological solutions for optimizing transport routes.

The article presents a mathematical model of optimal product delivery for restaurant business enterprises.

The purpose of the work is to present the parameters and algorithms of graph theory as a tool for analyzing and optimizing product delivery routes for the restaurant business.

Routing algorithms based on graphical description are considered the most optimal analysis method for developing optimal product delivery routes, which helps minimize enterprise costs. The availability of various algorithms for the development of graph bypass routes allows you to choose the most convenient option depending on transport capabilities, the type of goods and services, time constraints, etc. A scientific and methodological basis is proposed for the use of graph theory parameters and algorithms to optimize delivery routes, which helps minimize product costs in the restaurant business, taking into account the financial component. The use of the Bellman-Ford method of dynamic programming for this algorithm as a search for the shortest path in a weighted graph is substantiated. Two options for traversing the graph in depth for chain and cyclic routes were developed and the shortest routes for delivering products and raw materials from Kyiv to all cities where «Sushiya» establishments are located were determined, namely: Irpin, Bucha, Boryspil, Cherkasy, Vinnytsia, Lviv. The results of the work consist of practical recommendations on using a logistical approach in the service system to improve the quality of services and reduce costs, which directly affects the profitability and financial stability of the restaurant business.

Keywords: graph theory, modelling, restaurant, optimal route, supply, cost minimization, financial sustainability, logistics

JEL Classification: M31, C61

INTRODUCTION

Modern requirements for successful business and the ever-increasing level of competition require the development and implementation of marketing, logistics and other capabilities of the organization to meet customer requests and needs. Because of this, there is an urgent need for further research into the optimization of the company's transport logistics processes and the outline of prospects for their further development.

The choice of methods for finding the optimal route for the delivery of the company's products determines the provision of opportunities to reduce time and material resources. This helps reduce logistics costs, fuel costs, transport maintenance and labour costs, which has a positive impact on the financial results of the enterprise. The shortest path problem based on data structure has become a popular research topic in graph theory. "The purpose of graphs is to present too numerous or complex data for their adequate description in text or algorithm. Ensuring the clarity and correctness of the data description in the graphs ensure the efficiency of using the graphs" (Karkulovsky et al., 2023). As a result, the resulting shortest path is optimal. The relevance of the conducted research lies in the importance of using graph theory to find the optimal transportation route, which will help improve the efficiency of the transportation system, ensuring the competitive advantages of restaurant business enterprises. In addition, reducing logistics costs and increasing delivery speed help improve the company's financial stability, allowing for more flexible budget allocation and reinvesting the saved funds in service development or expanding the product range.

LITERATURE REVIEW

Analyzing scientific publications devoted to the solution of theoretical and applied problems of building optimal routes for the movement of vehicles of enterprises, as well as mechanisms of management of supply chains, the main areas of scientific research should be singled out.

Many applied problems from various spheres of human activity (communication, transport networks, etc.), in which it is necessary to operate with several connections between nodes, are adequately modelled with the help of multigraphs, that is, graphs in which several edges can exist between vertices. Time-dependent routing is the design of the "best" routes in the form of a graph in which the time to travel an arc can vary depending on the planning horizon (Gendreau et al., 2015).

Papers are devoted to the problems of mathematical modelling of federated machine learning, the purpose of which is the development of algorithms where the computing and communication load is balanced (Bahmani et al., 2017; Pan & Yang. 2010; Malinovsky, 2020). This technology will enable the widespread adoption of machine learning solutions, as well as flexible and manageable real-time data. In the logistics activities of companies, the use of devices and programs for automatic control of vehicles will allow to reduce the budget. Fedotova (2017) suggests coordinating such management objects in the logistics system as the interaction of chain participants, logistics functions, and business processes and creating a certain structure for them, determining the levels of integration.

Chuhrai (2009) claims that at the current stage, the market success of enterprises largely depends on improving their functioning in flexible supply chains, as well as on the effective use of resources on the way to achieving this flexibility. Tryfonova & Kravets (2019) noted that the markets for the sale of finished products determine the nature and volume of cargo flows, which are realized "with the synchronized operation of various types of transport, as well as the spatial concentration of transport and storage complexes, aimed at minimizing the costs associated with the transportation and storage of material resources" (Komov, 2022). The existing global practice of conducting business is characterized by a high level of integration and organization of business processes (Stebliuk et al., 2022; Shkolnyk, et al., 2019; Zhuravka, et al., 2023) related to the maintenance of cargo flow between all links of the transport and logistics chain (Ehmke et al., 2012; Zaverbnyi, 2018).

Scientists (Gardner et al., 2019; Khanin et al., 2019; Hryhorash et al., 2022) reveal the peculiarity of using the technology "supply chain transparency", which can be defined as the disclosure of information about the names of suppliers, the sustainability conditions of suppliers and the purchasing practices of buyers. From a logistics perspective, technology has enabled real-time tracking of deliveries and inventory, allowing managers to make informed decisions about resource allocation and route optimization.

1. Empirical studies conducted by scientists (Ali et al., 2023, Wang et al., 2022) emphasize, that graph theory serves as a foundational mathematical discipline that explores the complex interplay of graphs and diagrams as tools for visualizing and representing mathematical truths.
2. To solve the problems of optimizing transport routes, scientists Volosova (2023), Kavun (2014) propose the use of a mathematical model, namely in the form of a graph model, that is, using vertices and edges between them. The optimal route is understood as the route by which it is possible to deliver the products within the acceptable time frame with minimal transport costs, as well as preserving the consumer properties of the products (Datsko & Tsvir, 2016). Depending on the tasks and strategy of the company, the choice of transport for product delivery is carried out. At the same time, the location of production, and technical and economic features of various types of transport, which determine the spheres of their rational use, are taken into account.

The work (Huang et al., 2017) considers a route-path approximation method that generates almost optimal solutions for (path flexibility) TDVRP–PF under stochastic traffic conditions It is noted that the task of vehicle routing largely depends on the quality of the road network representation.

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This work (Menares et al., 2023**)** presents a bi-objective time-dependent vehicle routing problem with delivery failure probabilities (TDVRPDFP). Two objectives are jointly minimized: operational costs and delivery failure rates. Both travel times and costs, as well as the probabilities of delivery failure and service times and costs, are considered time-dependent.

One of the important typical tasks of logistics in the field of food is to determine the optimal location of a restaurant, warehouse, production and distribution centre and other elements of the logistics chain in the restaurant business (Smirnov, 2009). It has been shown that if a restaurant or chain has an efficient supply chain, they are able to deliver products with sufficient shelf life and create a quality end product. Shcherbina (2020), Stebliuk et al. (2022), Iraldo (2017) reveal the peculiarities of the hotel and restaurant business, their impact on logistics and supply chain management, determine the advantages of using a logistics approach and investments (Jereb, 2017) in the hotel and restaurant business.

The analysis of the scientific literature shows that the implementation of the interests of the subjects of logistics activity regarding the improvement of the optimization model of the enterprise's transport logistics processes requires the development of a mathematical model of the optimization of transport routes based on a graph representation (Gulyanytskyi & Pavlenko (2017)), Seredyuk (2014). The use of graph models can significantly increase the profitability of companies in stable markets, but in conditions of high uncertainty, it is necessary to combine algorithmic models with flexible strategic decisions. Algorithms based on graph theory help minimize transportation costs and reduce overall operating costs, which contributes to increasing the financial efficiency of the company.

Much attention is paid to approximate search methods, which include ant colony optimization algorithms. Danchuk & Svatko (2012) propose a method for solving the problem of finding the minimum path through the graph based on the application of a modified ant algorithm (finding the minimum costs for the delivery of goods according to the criterion of distances and transport times between nodes, depending on the total number of nodes).

The results of the research demonstrate the importance of using various optimization methods to increase the financial efficiency of enterprises by improving route management and logistics processes. The need to create a model for optimizing transport logistics has been identified, which will ensure the formation of optimal sizes of delivery batches, timely fulfilment of orders, reduction of logistics costs and strengthening the financial stability of the enterprise through the rational use of resources and minimization of total costs for transportation, storage and processing of products.

The analysis of the published works indicates the need for deepening scientific research aimed at developing a model for optimizing the company's transport logistics processes, which will affect the formation of optimal sizes of delivery lots, quality fulfilment of all orders within the established time frame, etc.

AIMS AND OBJECTIVES

The purpose of the article is to present the parameters and algorithms of graph theory as a tool for analyzing and optimizing product delivery routes for the restaurant business.

METHODS

To solve the tasks defined in the work, a set of general scientific and applied research methods was used, in particular: methods of scientific abstraction, induction and deduction, analysis and synthesis. Elements of graph theory were used to create the mathematical model, namely the Bellman-Ford method and system analysis.

The article considers models of optimal delivery of raw materials and products from Kyiv to all other cities where «Sushiya» restaurants operate, namely: Irpin, Bucha, Boryspil, Cherkasy, Vinnytsia, Lviv.

Mathematical formulation of the problem

Consider a weighted undirected connected graph, the weights of which are the distances between the cities that are vertices of the graph. This graph has 7 vertices and 14 edges. Table 1 shows the vertices of the graph, their degrees, the names of the cities they represent, as well as their neighbouring vertices. The diagram of the graph is shown in Figure 1.

Table 1. Adjacency and degree lists of graph vertices.

Vertice	City	Vertices of the graph adjacent to it	The degree of the vertice
V1	Kyiv	V2, V4, V6, V7	4
V2	Irpin	V1, V3, V6	3
V3	Bucha	V2, V6, V7	3
V4	Boryspil	V1, V5, V6, V7	4
V5	Cherkasy	V4, V6	3
V6	Vinnytsia	V1, V2, V3, V4, V5, V7	6
V7	Lviv	V1, V2, V3, V4, V6	5

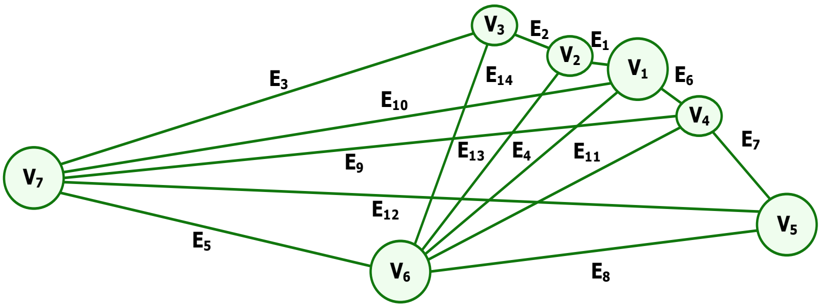

Figure 1. Graph diagram.

Table 2 lists the edges of the graph, and indicates which vertices they connect and their weights - distances (in km) between the corresponding cities.

Table 2. Lists of edges and their weights.

Edges	Weights of edges, km	Edges	Weights of edges, km
E1= V1V2	27	E8= V5V6	340
E2= V2V3	5	E9= V4V7	585
E3= V3V7	533	E10= V1V7	539
E4= V1V6	268	E11= V4V6	304
E5= V6V7	364	E12= V5V7	726
E6= V1V4	40	E13= V2V6	254
E7= V4V5	153	E14= V3V6	260

Powers of sets of vertices and edges of the graph:

$n = |V| = 7, \quad m = |E| = 14$

The rank $rang\,(G)$ of the graph G is a number equal to the difference between the number of its vertices *n=7* and the connectivity component *p=1:*

$rang(G) = 7 - 1 = 6.$

The cyclomatic number $\nu(G)$ of the graph G is equal to the difference between the number of its edges m and vertices *n*, to which is added the number *p* of the components of its connectivity:

$\nu(G) = m - n + p = 14 - 7 + 1 = 8.$

Let's build the incidence matrix M_i of this graph for the convenience of its further study:

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$$Mi = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The rows of the incidence matrix correspond to the vertices of the graph, and the columns correspond to the edges of the graph. The number of units in each row of the matrix is equal to the power of the corresponding vertex. The given graph is connected and simple since its adjacent vertices are connected by one edge. Since the graph has vertices of odd degrees, it is not an Euler graph, that is, it is impossible to organize an Euler cycle on it. For the convenience of performing the steps of the graph traversal algorithm, we present it in the form of the adjacency matrix M_s - a square matrix of the seventh order:

$$Ms = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

To obtain the optimal delivery route, we will use the graph traversal method, which is the basis of many other graph traversal methods. The depth-first search algorithm scans all the edges and all the vertices of the graph G once. During the search, the vertices of the graph G are successively assigned new numbers from 1 to n, and the edges receive marks of two classes: a direct edge $E(+)$or a reverse edge $E(-)$.

RESULTS

The article examines models of optimal delivery of raw materials and products from Kyiv to other cities where restaurants of the «Sushiya» company operate. «Sushiya» LLC is a chain of catering establishments of Japanese cuisine, which is part of one of the largest businesses in Ukraine - USG (Uncle Sam's Group) Holding. At the beginning of 2006, the first restaurant of Japanese cuisine of the Sushiya company was opened in the centre of Kyiv. There are 24 restaurants in 7 cities in Ukraine. The enterprise has a network organizational structure. This type of organizational structure is typical for restaurant chains with a single corporate style. The company has the following divisions: operations department (control of all facilities and delivery services), logistics department, finance department, marketing department, as well as information technology department and HR department. Figure 2 shows the main indicators of the restaurant's activity.

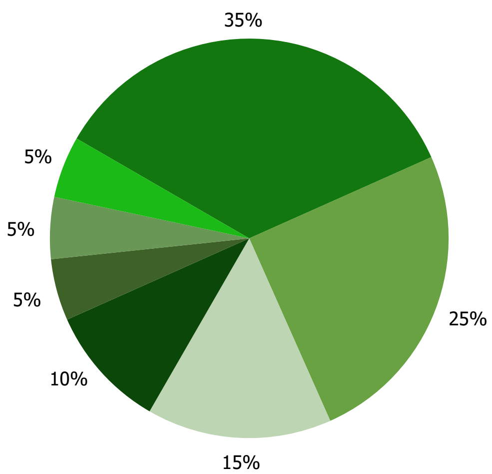

Figure 2. The main indicators of the restaurant's activity.

Main indicators: (food cost - 35%, profit - 25%, salary fund - 15%, rent - 10%, taxes, utility payments, business expenses - 5%). Rating of factors according to customer reviews (company website) (Table 3).

Table 3. Rating of factors affecting customer loyalty.

Factors	Rating, %
Assortment of dishes	4.20
The quality of prepared dishes	20.10
Comfort level	12.90
Loyalty program	3.90
Discounts	3.80
Convenient location	10.70
Additional Services	5.80
Quality of service	17.10
Price	16.60
Image	4.90

The main factors influencing the company's activities are listed in Table 4.

Table 4. Strengths and weaknesses of the company.

Factor	Strength	Weakness
A large number of restaurants, wide coverage of the regions of Ukraine	+	
Adaptation of the official website for mobile devices and availability of special applications for IOS and Android	+	
No bonus program		-
The assortment is not wide enough to cover the target segments		-
Effective sales promotion policy	+	
Independence from intermediaries	+	
The marketing mix is focused on only one segment of consumers (active youth)		-
The company is part of USG Holding, which provides it with financial support	+	
Favourable location of restaurants	+	
Inconsistency of the assortment policy of the segmentation strategy		-
The largest delivery service throughout Ukraine among restaurants of Japanese cuisine	+	
A system of motivation for restaurant staff has been established	+	
An established system of personnel training and team building, as a result of which staff turnover is low, high personnel qualification	+	
Using a "falling leader" pricing strategy for the most popular items in the assortment	+	
Using the innovative "Order man" guest service system	+	
Availability of own management school	+	
The same assortment in all restaurants does not make it possible to take into account differences in the needs of the target consumers of each restaurant		-
Dependence on USG Holding when making management decisions		-

The established logistics system of the company includes the purchase of raw materials from manufacturers for deliveries to restaurants. In fact, 99% of product items of the food group (excluding bar items) and 90% of items of the non-food group are subject to centralized procurement. The company strictly adheres to the established delivery schedule. It is also worth noting that the range of dishes throughout the «Sushiya» chain of restaurants is the same, regardless of the location of the establishment, differences in demand between different target audiences depending on the location of the restaurant, etc. are not taken into account. The product range of the «Sushiya» company is presented in Table 5.

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Table 5. The company's product range.

The company is a producer of rank	Depth of assortment	Number of positions
Sushi bar	Roles "Philadelphia"	11
	Roles "California"	5
	Maki-rolls	3
	Futomaki	2
	Corporate roles	5
	"Dragons"	3
	Warm rolls	2
	Sushi	8
Kitchen	Soups	8
	Salads	6
	Appetizer	4
	Hot meals	13
	Desserts	12
Bar	**Soft drinks**	
	Tea	7
	Coffee	7
	Juices	6
	Mineral and sweet water	7
	Cocktails	6
	Alcohol	
	Beer	7
	Wine	12
	Strong alcohol	15
	Cocktails	10

Wide coverage of the regions of Ukraine is the strength of Sushiya restaurants compared to other main competitors, which are concentrated only in the territory of Kyiv. In addition to the chain of restaurants, the Sushiya company consists of a head office located in Kyiv, as well as a central warehouse, where all products and ingredients are received from suppliers, and a call centre.

On the basis of graph theory, we will consider the mathematical model of the transport network and algorithms for finding the optimal route.

According to the mathematical statement of the problem, we will consider the **1st variant of traversing the graph.**

1. We start from the top of V_1, put the number 1 in accordance with it.
2. We choose an arbitrary vertex adjacent to V_1, for example, V_2, we assign the number 2 to it, and the edge E_1 is straight and receives a (+) mark.
3. We choose an arbitrary vertex adjacent to V_2, for example, V_3 (Bucha), number 3 corresponds to it, and edge E_2 has a (+) mark.
4. The vertex V_3 is adjacent to the vertex V_7, we assign it the number 4, and the edge E_3 connecting them is straight.
5. Vertex V_7 is one of the adjacent new vertices that have not yet been renumbered, there is vertex V_6, which we assign the number 5, and the edge E_5 has a (+) mark.
6. Vertex V_6 has a new vertex V_5 adjacent to it, we select it and give it the next number 6, and the edge E_8 is straight.
7. We choose the vertex adjacent to V_5, which does not yet have a new number, this is the vertex V_4, we assign it the number 7, and the edge E_7 is straight and is marked (+).
8. Since all vertices are renumbered, the deep traversal of the graph is complete.

Thus, the core of the graph consists of straight edges E_1, E_2, E_3, E_5, E_8, E_7.

All other edges are reversed and make up the chords of the graph, their number is 8, which is equal to the cyclomatic number of the graph, which indicates the correctness of the performed traversal of the graph.

Since all vertices are renumbered, the deep traversal of the graph is complete. A simple chain $V_1 – V_2 – V_3 –V_7 –V_6 – V_5 – V_4$ is obtained, bypassing all vertices of the graph. The length of this route is equal to the sum of the weights of its edges $W(E_1 +E_2 +E_3 +E_5 +E_8+E_7)$=1422 km.

Since the vertex V_4 is adjacent to the initial vertex V_1, it is possible to create a simple cycle $V_1 – V_2 – V_3 –V_7 –V_6 – V_5 – V_4 – V_1$, after which you can return to the initial vertex, and its length is $W(E_1 +E_2 +E_3 +E_5 + E_8 +E_7+ E_6)$=1462 km.

A second variant of traversing this graph in depth is possible, which forms the following simple chain $V_1– V_4 – V_5 – V_6 – V_2 – V_3 – V_7$, which also traverses all vertices of the graph. In this version, the core of the graph consists of straight edges E_6, E_7, E_8, E_{13}, E_2, E_3. All other edges are reversed and are chords of the graph, there are 8 of them, i.e., their number is equal to the cyclomatic number, so this variant of traversing the graph is also correct.

The length of this route is $W(E_6+E_7+E_8+E_{13}+E_2+E_3)$=1325 km. The vertex V_7 is also adjacent to the initial vertex V_1, that is, it is possible to create a simple cycle $V_1 – V_4 – V_5 – V_6 –V_2 – V_3 – V_7 – V_1$ from this simple chain, after which you can return to the initial vertex, and its length is $W(E_6+E_7+ E_8+E_{13}+E_2+E_3+E_{10})$=1864 km.

The results of the obtained two variants of traversal of the graph in-depth and the simple cycles built on them are shown in Table 6.

Table 6. Product delivery routes.

Version	Simple chain	Distance, km	Simple cycle	Distance, km
1	V1 – V2 – V3 –V7 –V6 – V5 – V4	1422	V1 – V2 – V3 –V7 –V6 – V5 – V4 – V1	1462
2	V1 – V4 – V5 –V6 –V2 – V3 – V7	1325	V1 – V4 – V5 –V6 –V2 – V3 – V7 – V1	1864

Thus, if it is necessary to deliver products for restaurants after centralized purchases from Kyiv to restaurants in the country and return to Kyiv, i.e. to carry out a circular route, then it is better to choose the first option, according to which the distance of the entire delivery route is equal to 1462 km, and if it is necessary to carry out the transportation of raw materials and products to restaurants in 7 cities along the chain, it is better to choose the second option of the route, which is 1325 km.

If there are trips to deliver raw materials from Kyiv to a certain city, it is convenient to use the Bellman-Ford algorithm to determine the shortest paths to develop optimal routes. This algorithm is used when it is necessary to determine the shortest routes from one vertex of the graph, in our problem from the vertex V1, which is the input, to all other vertices in the weighted graph.

The working principle of the Bellman-Ford algorithm is to first determine the lengths of the shortest paths from the Vs - source vertex to other vertices provided that they contain no more than one edge, then the metrics of the shortest paths provided that they contain no more than 2 ribs, etc. The shortest path that consists of no more than K edges is called the shortest K-path.

Let $D_s^{(K)} = 0$ be the metric of the shortest K-path from the vertex V_s to the vertex V_i. Let us assume that for the vertex Vs the metric $D_s^{(K)} = 0$ for all K.

A step-by-step description of the Bellman-Ford algorithm looks like this. At the initial step of the algorithm (K =0) is accepted $D_i^{(0)} = \infty$ to all $i \neq s$.

At the following steps of the algorithm (for each K >1), the Bellman-Ford iteration is performed for all:

$$D_i^{(K)} = min\left[D_i^{(K-1)},\ D_j^{(K-1)} + l_{j,i}\right], \quad (1)$$

where $l_{j,i}$ is the weight of the edge between the vertices V_j and V_i.

The algorithm ends when the value of K reaches the number of the last vertex Vn, that is, in the worst case, the tree of the shortest paths has the form of a chain with a length of (N -1) edges. Let's apply the described algorithm:

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1. K=0:

$$D_1^{(0)} = min\left[D_2^{(0)}, D_1^{(0)} + l_{1,2}\right] = min[\infty, 27] = 27; D_2^{(0)} = min\left[D_4^{(0)}, D_1^{(0)} + l_{1,4}\right] = min[\infty, 40] = 40; D_3^{(0)} = min\left[D_7^{(0)}, D_1^{(0)} + l_{1,7}\right] = min[\infty, 539] = 539; D_4^{(0)} = min\left[D_6^{(0)}, D_1^{(0)} + l_{1,6}\right] = min[\infty, 340] = 268.$$

2. We choose the vertex V2, the weight of which is the smallest. We perform the next iteration. K=1:

$$D_5^{(1)} = min\left[D_3^{(0)}, D_2^{(0)} + l_{2,3}\right] = min[\infty,\ 27+5] = 32; D_6^{(1)} = min\left[D_6^{(0)}, D_2^{(0)} + l_{2,6}\right] = min[340,\ 27+254] = 281.$$

3. We choose the vertex V3, the weight of which is the smallest. We perform the next iteration. K=2:

$$D_7^{(2)} = min\left[D_7^{(0)}, D_5^{(1)} + l_{3,7}\right] = min[539,\ 32+533] = 539; D_8^{(2)} = min\left[D_4^{(0)}, D_5^{(1)} + l_{3,6}\right] = min[268,\ 32+260] = 268.$$

4. We choose the vertex V6, the weight of which is the smallest. K=3:

$$D_9^{(3)} = min\left[D_7^{(0)}, D_8^{(2)} + l_{6,7}\right] = min[533,\ 268+364] = 533; D_{10}^{(3)} = min\left[D_2^{(0)}, D_8^{(2)} + l_{6,4}\right] = min[40,\ 268+304] = 40; D_{11}^{(3)} = min\left[D_5^{(0)}, D_8^{(2)} + l_{6,5}\right] = min[\infty,\ 268+340] = 608.$$

5. We select the vertex V4 and perform the following iteration. K=4:

$$D_{12}^{(4)} = min\left[D_5^{(0)}, D_{10}^{(3)} + l_{4,5}\right] = min[\infty,\ 40+153] = 193; D_{13}^{(4)} = min\left[D_7^{(0)}, D_{10}^{(3)} + l_{4,7}\right] = min[539,\ 40+585] = 539.$$

6. We choose the vertex V5, the weight of which is the smallest. We perform the next iteration. K=5:

$$D_{14}^{(5)} = min\left[D_7^{(0)}, D_{12}^{(4)} + l_{5,7}\right] = min[539,\ 193+726] = 539.$$

All vertices have been passed, so the algorithm is complete. Table 7 lists the minimal paths to all vertices from the vertex V_1.

Table 7. Shortest paths from vertex V_1 to other vertices.

The vertices that the route connects	Route	Distance, km
V1 –V2 (Kyiv-Irpin)	V1 –V2 (Kyiv-Irpin)	27
V1 –V3 (Kyiv-Bucha)	V1 –V2 –V3 (Kyiv-Irpin-Bucha)	32
V1 –V4 (Kyiv-Boryspil)	V1 –V4 (Kyiv-Boryspil)	40
V1 –V5 (Kyiv-Cherkasy)	V1 –V4 –V5 (Kyiv-Boryspil-Cherkasy)	193
V1 –V6 (Kyiv-Vinnytsia)	V1 –V6 (Kyiv-Vinnytsia)	268
V1 –V7 (Kyiv-Lviv)	V1 –V7 (Kyiv-Lviv)	539

Thus, the shortest routes for the delivery of products and raw materials from Kyiv - the top of V_1 to other cities where Sushiya establishments are located - were obtained. Solving problems using the method based on the Bellman-Ford optimality principle is that at each stage it is possible to take into account previous changes and manage the course of events to make management decisions in order to optimize logistics chains. Its further functional capacity, determination of its development prospects in accordance with the chosen logistics strategy depends on the correctly chosen geographical position of the object. The latter is aimed at:

- cost minimization;
- optimization of time parameters;
- effective customer service;
- high quality;
- the flexibility of the volume of products offered;
- application of new technologies;
- geographical location.

Well-established logistics processes provide not only financial benefits but also improve the quality of services, which has a positive impact on the reputation of restaurant establishments (Table 8).

Table 8. Financial benefits of optimizing delivery in the restaurant business.

Factors	Description
Reducing costs for transportation and storage of products	Reducing costs for product transportation and storage. Rationalization of logistics processes, optimization of vehicle loading for product delivery, and efficient use of warehouse space contribute to reducing costs for storing ingredients and transporting supplies
Acceleration of the order delivery process	The use of modern order management systems and optimization of delivery routes allows you to reduce the delivery time of products and orders, improving customer service and reducing logistics costs
Improving the efficiency of interaction with suppliers	Planning product deliveries and timely replenishment of stocks reduces the risk of delays, which allows you to maintain a continuous service process in the restaurant
Optimization of logistics routes for delivery	Reduces transportation costs and increases restaurant efficiency by ensuring timely receipt of necessary ingredients
Improving restaurant logistics	Provides a high level of guest service, improving their experience and increasing loyalty, which contributes to increased sales

The obtained results of the research can be used in the restaurant business for logistics planning and modelling of the search for the optimal transportation route, which will help to solve the complex tasks of managing the supply chains of the company's products.

DISCUSSION

The power of graphs goes beyond simple visual representation, allowing the creation of complex network architectures through the use of various graph operations. Calculating the most efficient route from the point of departure to the destination in the road network is a key component of both car navigators and logistics services of companies.

The issue of developing optimal logistics routes using graph models is driven by the desire to reduce the costs of various groups associated with the movement of goods to increase the efficiency of transportation and reduce the risk of delays in the delivery of material resources. Various graph traversal algorithms have been developed to address this issue. Steinhardt (2006) concluded that Dijkstra's algorithm specializes in finding the shortest paths between the vertices of a graph.

Graph theory advocates emphasize that the use of Dijkstra's algorithm in combination with hierarchical network modelling has reduced transportation costs by 28% compared to traditional methods Li, M., & Yang, S. (2011).

Andrew V. Goldberg (2008) came to similar conclusions, studying vertex-to-vertex shortest path algorithms using the depth-first search algorithm, which is the basis for many other graph traversal algorithms. Abbasi et al. (2011) considered the dynamic problem of finding the shortest path, which is used in dynamic flows of minimum costs for the transformation problem. Babich V. et al. (2023) proved that the Floyd–Warshell algorithm is effective in calculating all shortest paths in dense graphs when there are many pairs of edges between pairs of vertices, and in the case of sparse graphs, the Dijkstra algorithm is more effective. The Bellman-Ford algorithm, highlighted in the research work, calculates the shortest paths from one vertex to all others in a weighted graph (like the Dijkstra algorithm). It can work on graphs that have edges with negative weights. However, it cannot be applied to all such graphs, since each subsequent pass along the path consisting of edges with a negative sum of weights only improves the result. To find all the shortest paths from vertex s to all others using the Bellman-Ford algorithm, you need to use the dynamic programming method: break the problem into subproblems and find their solution. The solution to each such subproblem is to determine the shortest path from one individual vertex to another.

In these works, scientists propose certain algorithms for specific classes of problems, which necessitates their combination depending on the type of route and the characteristics of the problem. This allows combining different algorithms to effectively solve routing problems in logistics and delivery, taking into account specific factors and financial constraints.

This is consistent with the conclusions of Ru, S., who proposes the use of combined algorithms for optimizing intermodal vehicle routes in logistics. The authors Zhang, X., & Li, Y. (2023) built a distribution route optimization model that uses a hybrid intelligent algorithm to minimize costs and optimize delivery routes.

Also, research by scientists confirms that optimizing logistics costs and routing have a direct positive impact on return on

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assets (ROA) and return on investment (ROI). However, profitability depends on the correct choice of logistics model (Huang & Siao, 2023).

But despite sophisticated graph models, companies in unpredictable conditions suffer financial losses due to global supply disruptions, proving that algorithms cannot always predict crisis scenarios (Atayah et al., 2021). Therefore, two delivery modes have been proposed to solve the problems: the modern delivery mode of chain restaurants (MDMCR) and the improved delivery mode of chain restaurants (IDMCR). IDMCR is a one-step approach to directly solve the problem of delivery routing for multiple branches (Cao, Q., Fu, C., & Li, H., 2022). Some researchers insist on its financial advantages, while others warn of the hidden risks and limitations of this technology. Graph algorithms become most effective when combined with other technologies, such as geographic information systems (GIS) or machine learning for demand forecasting.

Most analysts (Bai et al., 2018) express the opinion that for the greatest accuracy of calculations, it is advisable to represent the problem of vehicle routing with time dependence since then significant savings are achieved in terms of costs and fuel consumption due to the existing consideration of route flexibility.

We consider it necessary to pay attention to solving the problem of forecasting the route of export and delivery using a dynamic spatiotemporal graph-based model called Graph2Route (Wen, H., Lin, Y., Mao, X., Wu, F., Zhao, Y., Wang, H., Zheng, J., Wu, L., Hu, H., & Wan, H., 2022). This model uses the basic graph structure and functions in the encoding and decoding process. In addition, the dynamic nature of graphs can spontaneously describe the dependencies between different types of developing problems, including financial ones.

Currently, research is underway into models and algorithms for solving practical problems of finding optimal paths, taking into account existing limitations, given that traditional methods for finding the optimal shortest path often cannot be used for real-time applications due to their computational complexity.

Therefore, route optimization becomes a critical task for ensuring the financial sustainability of businesses. In this context, graph theory offers a wide range of solutions, but their effectiveness in real conditions remains a subject of scientific debate.

CONCLUSIONS

In the simulated situation of laying a route for a restaurant business company, an algorithm was obtained, which was determined as the best way to determine the optimal paths in different spaces. The existing mathematical apparatus of graph theory can be used for analysis, optimal construction and development of networks. Based on graph theory, the mathematical model of the transport network and algorithms for finding the optimal route are considered. Graph theory provides a simple but effective solution to the given problem.

The practical significance of the model being built is determined by the fact that its application will allow to reveal the actually existing relationships and provide an optimal solution to the given problem in the presence of certain conditions with little computational complexity.

The proposed algorithms for finding the optimal or minimum path between given vertices on a known graph make it possible to reduce transportation costs and increase the financial stability of companies.

Priority areas of research should be directed towards the development of adaptive algorithms that learn in real-time and take into account unpredictable factors that affect the delivery process. The integration of several graph algorithms can significantly improve the financial results of the restaurant business, provided that real operational risks are taken into account. Establishments should carefully assess the potential financial effect and initial costs, especially when solving complex logistical problems, such as optimizing delivery routes in conditions of limited resources and the number of service points.

As for management implications, this mathematical model can be used to determine future strategies and technologies for the implementation of certain logistical tasks, as well as for the development of new technological solutions for current and future needs. The obtained results form the basis for further research in the direction of optimization of transportation for justified management decisions in the field of strategic management of restaurant business enterprises.

ADDITIONAL INFORMATION

AUTHOR CONTRIBUTIONS

All authors have contributed equally.

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CONFLICT OF INTEREST

The Authors declare that there is no conflict of interest.

REFERENCES

1. Gendreau, M., Ghiani, G., & Guerriero, E. (2015). Time-dependent routing problems: A review. *Computers & Operations Research, 64,* 189-197. http://dx.doi.org/10.1016/j.cor.2015.06.001
2. Bahmani, R., Barbosa, M., Brasser, F., Portela, B., Sadeghi, A., Scerri, G., & Warinschi, B. (2017). Secure Multiparty Computation from SGX. *In Financial Cryptography and Data Security - 21st International Conference, FC 2017. Sliema, Malta.* Revised Selected Papers, 477–497. https://doi.org/10.1007/978-3-319-70972-7_27
3. Karkulovskyy, V., Kryvyy, R., Jaworskyi, N., & Bezkostyi, A. (2023). A Software Complex for Researching Algorithms for Working with Graphs*. 17th International Conference on the Experience of Designing and Application of CAD Systems (CADSM),* Jaroslaw, Poland, 2023. 1-5. https://doi.org/10.1109/CADSM58174.2023.10076530
4. Komov, M. (2022). Factors and Peculiarities of the Formation of the Single Transport Space in the Eurasian Economic Union (EAEU). Transportation Research Procedia, 63, 753-758. https://doi.org/10.1016/j.trpro.2022.06.071
5. Pan, S., & Yang, Q. (2010). A Survey on Transfer Learning. *IEEE Transactions on Knowledge and Data Engineering, 22*(10), 1345–1359. http://dx.doi.org/10.1109/TKDE.2009.191
6. Malinovsky, G., Kovalev, D., Gasanov, E., Condat, L., & Richtárik, P. (2020). From local SGD to local fixed-point methods for federated learning. Cornell University. https://arxiv.org/abs/2004.01442v2
7. Fedotova, I. V. (2017). Conceptual bases of integrated logistics. Economics of the transport complex, 30, 15-32. https://doi.org/10.30977/ETK.2225-2304.2017.30.0.15
8. Chukhrai, N. I. (2009). Otsiniuvannia funktsionuvannia lantsiuha postavok: sutnist ta kontseptualni pidkhody. Visnyk Natsionalnoho universytetu «Lvivska politekhnika», 647, 276 - 285. https://vlp.com.ua/files/48_5.pdf
9. Tryfonova, O.V., & Kravets', O.Yu. (2019). Formation of optimal logistics systems in the process of strategic management of sustainable development of the enterprise, *Ekonomichnyj prostir, 142*, 217—226. http://srd.pgasa.dp.ua:8080/bitstream/123456789/2586/1/Tryfonova.pdf
10. Stebliuk, N., Volosova, N., Koberniuk, S., & Rybak, O. (2022). Strategic Management of Factories in Conditions of Innovation-Marketing Orientation in the Industrial Market. *International Journal of Information Technology Project Management (IJITPM), 13*(2), 1–16. http://dx.doi.org/10.4018/IJITPM.311848
11. Zhuravka, F., Nebaba, N., Yudina, O., Haponenko, S., & Filatova, H. (2023). The hospitality market in Ukraine: War challenges and restoration possibilities. *Innovative Marketing, 19*(1), 140-150. https://doi.org/10.21511/im.19(1).2023.12
12. Shkolnyk, I., Pisula, T., Loboda, L., & Nebaba, N. (2019). Financial crisis of real sector enterprises: an integral assessment. *Investment Management and Financial Innovations, 16*(4), 366-381. https://doi.org/10.21511/imfi.16(4).2019.31
13. Ehmke, J.F., Steinert, A., & Mattfeld, D.C. (2012). Advanced routing for city logistics service providers based on time-dependent travel times. *Journal of Computational Science, 3*(4), 193-205. http://dx.doi.org/10.1016/j.jocs.2012.01.006
14. Zaverbnyi, A.S. (2018). Problems and prospects of forecasting in the management system of sales and supply activities in the languages of European integration. *Economic Journal of Odessa Polytechnic University, 3*(5), 13–19. https://economics.opu.ua/ejopu/2018/No3/13.pdf
15. Gardner, T., Benzie, M., Börner, J., Dawkins, E., Fick, S., Garrett, R., & Wolvekamp, P. (2019). Transparency and sustainability in global commodity supply chains. *World Development, 121*, 163-177. http://doi.org/10.1016/j.worlddev.2018.05.025
16. Khanin, I., Shevchenko, G., Bilozubenko, V., & Korneyev, M. (2019). A cognitive model for managing the national innovation system parameters based on international comparisons (the case of the EU countries). *Problems and Perspectives in Management, 17*(4), 153-162. https://doi.org/10.21511/ppm.17(4).2019.13
17. Hryhorash, O., Bocharov, D., Korneyev, M., Rudyanova, T., & Hryhorash, T. (2022). The quality of higher education

and its funding in countries with different levels of socio-economic development. *Knowledge and Performance Management, 6*(1), 49-61. https://doi.org/10.21511/kpm.06(1).2022.05

18. Ali, N., Kousar, Z., Safdar, M., Tolasa, F.T., & Suleiman, E. (2023). Mapping Connectivity Patterns: Degree-Based Topological Indices of Corona Product Graphs. *Journal of Applied Mathematics,* 8975497. https://doi.org/10.1155/2023/8975497
19. Wang, Shun, Lv, Yimei, Peng, Yuan, Piao, Xinglin, & Zhang, Y. (2022). Metro Traffic Flow Prediction via Knowledge Graph and Spatiotemporal Graph Neural Network. *Journal of Advanced Transportation,* 2348375. https://doi.org/10.1155/2022/2348375
20. Volosova, N., & Belobrov, Y. (2023, May 16 – 19). Application of graph theory for solving logistics problems. The 19th International scientific and practical conference «Innovative approaches to solving scientific problems». Tokyo, Japan. International Science Group. P.340-341.
21. Kavun, S. V., Mykhalchuk, I. V., Kalashnykova, N. I., & Zyma, O. G. (2012). A Method of Internet-Analysis by the Tools of Graph Theory. Intelligent Decision Technologies. *Smart Innovation, Systems and Technologies, 15*(1), 35–44. http://dx.doi.org/10.1007/978-3-642-29977-3_4
22. Datsko, M.V., & Tzvir, L.R. (2016). Construction of transport routes in logistics. *Scientific Bulletin of Kherson State University, 16*(4), 152-155. http://www.irbis-nbuv.gov.ua/cgi-bin/irbis_nbuv/cgiirbis_64.exe?I21DBN=LINK&P21DBN=UJRN&Z21ID=&S21REF=10&S21CNR=20&S21STN=1&S21FMT=ASP_meta&C21COM=S&2_S21P03=FILA=&2_S21STR=Nvkhdu_en_2016_16%284%29__39
23. Huang, Yixiao, Zhao, Lei, Woensel, Tom Van, & Gross, Jean-Philippe. (2017). Time-dependent vehicle routing problem with path flexibility. *Transportation Research Part B: Methodological, 95,* 169-195. http://dx.doi.org/10.1016/j.trb.2016.10.013
24. Menares, F., Montero, E., Paredes-Belmar, G., & Bronfman, A. (2023). A bi-objective time-dependent vehicle routing problem with delivery failure probabilities. *Computers and Industrial Engineering, 185*, 109601. https://doi.org/10.1016/j.cie.2023.109601
25. Smirnov, I. G. (2009). Restaurant logistics: international and Ukrainian aspects. *Foreign trade: law and economics, 4*(45), 4-18.
26. Shcherbina, V.V. (2020). Peculiarities of logistics processes in the hotel and restaurant business. *Development of methods of management and management in transport: Coll. of science Working, 1*(70), 99-114. https://doi.org/10.31375/2226-1915-2020-1-99-114
27. Stebliuk, N., Volosova, N., Nebaba, N., Yudina, O., Korneyev, M., & Zhuravka, F. (2022). Economic trends forecasting in the development of hotel business enterprises. *Financial and Credit Activity Problems of Theory and Practice, 5*(46), 182–193. https://doi.org/10.55643/fcaptp.5.46.2022.3876
28. Iraldo, F., Testa, F., Lanzini, P., & Battaglia, M. (2017). Improving Competitiveness for Hotels and Restaurants. *Journal of Small Business and Entrepreneurship Development, 24*(3), 607–628. https://doi.org/10.1108/JSBED-12-2016-021
29. Jereb, B. (2017). Mastering logistics investment management. *Transformations in Business and Economics, 16*(1 (40)), 100–120. https://www.researchgate.net/publication/316275989_Mastering_logistics_investment_management
30. Seredyuk, V. B. (2014). Application of economic and mathematical methods for solving economic problems. *Herald of socio-economic research, 1,* 69-73.
31. Gulyanitsky, L.F., & Pavlenko, A.I. (2017). Modelling time-dependent optimal routing problems: a review. *Mathematical modelling in economics*, 1-2. http://dspace.nbuv.gov.ua/bitstream/handle/123456789/131908/08-Hulianytskyi.pdf?sequence=1
32. Danchuk, V.D., & Svatko, V.V. (2012). Optimization of the path search on a graph in the dynamic problem of a traveling salesman by the method of a modified ant algorithm. System. research and information technologies, 2, 78-86. https://ela.kpi.ua/server/api/core/bitstreams/4ada56d6-941d-426a-a989-63a34a7b1779/content
33. The official website of the Sushiya company (n.d.). https://www.sushiya.ua/

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ЗАСТОСУВАННЯ ТЕОРІЇ ГРАФІВ ДЛЯ ОПТИМІЗАЦІЇ МАРШРУТІВ ДОСТАВКИ ПРОДУКЦІЇ ТА МІНІМІЗАЦІЇ ВИТРАТ У РЕСТОРАННОМУ БІЗНЕСІ

Важлива роль у системі управління підприємств ресторанного бізнесу в умовах конкуренції належить виконанню складних завдань управління ланцюгами постачань продукції та розробці нових технологічних рішень оптимізації транспортних маршрутів.

У статті подано математичну модель оптимальної доставки продукції для підприємств ресторанного бізнесу.

Метою роботи є представлення параметрів та алгоритмів теорії графів як інструмента для аналізу й оптимізації маршрутів доставки продукції для ресторанного бізнесу.

Алгоритми маршрутизації, засновані на графічному описі, вважають найбільш оптимальним методом аналізу для розробки оптимальних маршрутів доставки продукції, що сприяє мінімізації витрат підприємства. Наявність різноманітних алгоритмів розробки графів об'їзних маршрутів дозволяє обрати найбільш зручний варіант залежно від транспортних можливостей, видів товарів і послуг, часових обмежень тощо.

У статті запропоновано науково-методичне підґрунтя для використання параметрів та алгоритмів теорії графів із метою оптимізації маршрутів доставки, що сприяє мінімізації витрат продукції в ресторанному бізнесі, враховуючи фінансову складову. Обґрунтовано використання методу динамічного програмування Беллмана-Форда для цього алгоритму як пошуку найкоротшого шляху у зваженому графі.

Розроблено два варіанти обходу графа вглиб для ланцюгового та циклічного маршрутів і визначено найкоротші маршрути доставки продуктів і сировини з Києва до всіх міст, у яких розташовані заклади «Сушия», а саме: Ірпінь, Буча, Бориспіль, Черкаси, Вінниця, Львів.

Сформовано рекомендації щодо використання логістичного підходу в системі обслуговування для підвищення якості послуг, зниження витрат, що безпосередньо впливає на рентабельність і фінансову стійкість ресторанного бізнесу.

Ключові слова: теорія графів, моделювання, ресторан, оптимальний маршрут, постачання, мінімізація витрат, фінансова стійкість, логістика

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