ABSTRACT

Forecasting external public debt under conditions of uncertainty is important as it allows the country to respond adequately to economic and financial challenges, promotes efficient management of financial resources, formation of a stable financial policy and ensures the country's external debt security, which are critical elements for ensuring economic sustainability and sustainable development. The article's main purpose is to critically analyze and apply existing time-series forecasting methodologies to determine the future values of Ukraine's external public debt in conditions of uncertainty caused by the still unresolved consequences of the COVID-19 pandemic and Russia's invasion and full-scale war in Ukraine. Using three forecasting methods, namely trendline extrapolation, exponential smoothing, and autoregressive and moving average models, the paper forecasts the volume of external public debt until 2029 and presents a graphical representation of the debt dynamics from 2011 to 2029. The most pessimistic forecast for the growth of external public debt was revealed when applying the method of data extrapolation based on the trendline. A comparative analysis of the forecast values for the three forecasting methods has revealed common trends in the growth of public debt, as well as the key advantages and disadvantages inherent in each model. Importantly, the article emphasizes the common risks identified in forecasting Ukraine's external debt using time series analysis models, including the problem of achieving only short-term forecasting accuracy and insufficient flexibility taking into account complex and unexpected changes that may arise in conditions of uncertainty and economic instability. The results of the study provide valuable information for policymakers and stakeholders trying to navigate the complexities of managing external public debt under uncertainty.

Keywords: external public debt, forecasting, uncertainty, public debt management, economic development, Ukraine, wartime

INTRODUCTION

By February 2022, Ukraine's external public debt had already caused serious concern and was the subject of attention both domestically and internationally. The total amount of liabilities to foreign creditors increased, and the share of external debt in relation to the country's GDP increased, raising questions about external debt security. The current situation in Ukraine has only worsened. In contemporary reality, which is associated with the still unresolved consequences of the COVID-19 pandemic, Russia's full-scale war against Ukraine, new cascading reactions are emerging in most areas of activity.

Nowadays Government expenditures exceed UAH 250 billion per month, more than half of which are spent on defense. At the same time, the state's own budget revenues are not enough to cover even military expenditures, not to mention important areas such as social, medical, and other sectors. It is the external public debt that is now a tool for solving crucial problems and one of the key sources of budget revenues. Since February 24, 2022, the external public debt has increased by USD 35,576 billion. This amounted to 68% of the total public debt (as of October 30, 2023). (Ministry of Finance of Ukraine, 2023).

It is important to keep in mind that an excessive increase in external public debt, especially in wartime, can cause a series of problems for the country's economic stability.
That is why the development and implementation of an effective external public debt management system is crucial. One of its key tools is systematic forecasting, which allows to adequately assess possible risks, and plan strategies to reduce the debt burden and promote the stability of the financial system. Forecasts not only shape sound fiscal policy but also enable governments to respond proactively to new challenges. A well-built forecasting system becomes an important tool for ensuring sustainable economic development and preventing crises associated with excessive growth of external public debt. At the same time, inaccurate forecasts of external public debt can lead to serious consequences for the country's financial stability and economic development. Unpredictable changes in the amount of debt obligations may lead to an increase in financial pressure, particularly due to higher debt service and interest rates. This, in turn, may lead to a reduction in the ability to implement other essential programs, investments and social projects, which threatens the country's economic stability and overall welfare. Also, inaccurate forecasts make it challenging to plan strategies to reduce the debt burden, restore the country, help the victims, etc.

In general, effective financial management under uncertainty requires promptness and flexibility in adjusting budget strategies to consider unforeseen circumstances.

In this context, it is essential to take into account the use of external public debt forecasts. In general, effective financial management under uncertainty requires promptness and flexibility in adjusting budget strategies to take into account unforeseen circumstances.

In this context, it is important to consider that the use of external public debt forecasting models based on considering its relationship with other economic indicators can sometimes hinder the efficiency of management decision-making. For example, building VAR or ARMAX models based on a country's GDP may be a task that requires significant computational resources and time (because, first, a country's GDP is an annual indicator, and second, a substantial amount of data is necessary to build such models and, in particular, to make accurate forecasts). However, given the importance of accuracy, and most importantly, efficiency, it is important to focus on using forecasting models based on data from previous periods. This approach allows the model to be effectively adapted to changing conditions and make forecasts based on available data while ensuring greater efficiency in solving management tasks.

That is why it is important to study modern methods for forecasting external public debt data based solely on historical data, focusing on Ukraine, under conditions of uncertainty.

LITERATURE REVIEW

The scientific literature on external public debt forecasting under uncertainty covers various studies that use different methodologies and perspectives to address this complex issue.

External public debt management is an essential aspect of economic development policy, and its effective forecasting is necessary to ensure financial stability and sustainability.

Several scholars (e.g., Reinhart and Rogoff (2009), Zou et al. (2023), Aiyedogbon et al. (2022)) have contributed to the understanding of external public debt management, emphasizing the importance of debt security and efficient resource allocation. In particular, Reinhart and Rogoff (2009) conducted a seminal study analyzing the history of debt crises and emphasizing the cyclical nature of debt accumulation. Their work laid the foundation for further research on the factors that influence the dynamics of external public debt.

Subsequently, Sturzenegger and Zettelmeyer (2007) deepened the study of sovereign debt crises by focusing on the dynamics of creditor losses and the effectiveness of debt relief measures. The study delved into the complex interactions between debtor countries and their creditors in times of financial distress, providing information on the outcomes and consequences of different debt management strategies. These general studies provide a fundamental understanding of the dynamics of external public debt and lay the groundwork for studying debt management under uncertainty.

The risk management issue in the face of uncertainty is also addressed in scientific works. Risk management models and strategies to reduce financial risks in the context of public debt are studied from the perspective of the need for effective management of the country's financial resources (Chirume (2023) and Makedon and Korneyev (2014)). For example, Jaramillo et al. (2017) studied the phenomenon of debt surges, blind spots, and financial stress. Their study provides an in-depth analysis of the consequences of a sudden increase in debt, identifying potential blind spots and points of financial stress.

Filatova et al. (2023) delve into the realm of public policy and financial regulation with a focus on preventing and combating financial fraud. The article employs a bibliometric analysis to assess the current state of research in this domain. While the
primary focus of this work is different from the forecasting of external public debt, it contributes valuable insights into the regulatory environment and financial policy factors that play a crucial role in shaping a country's economic landscape.

Petrushenko et al. (2022) focused on assessing the impact of external debt on the country's economic development indicators using Ukraine as an example. This study examined the specific consequences of rising external debt for economic development at the beginning of the military conflict.

The link between external public debt and uncertainty is particularly pronounced during military conflicts. Eichengreen and Portes (1986) and Gontareva et al. (2022) investigated the relationship between war and debt, noting the tendency for borrowing to increase during conflicts. This paper highlights the complex relationship between geopolitical events, debt accumulation, and subsequent economic problems.

Drawing on the challenges posed by war, Collier and Hoeffler (2004) and Filatova et al. (2022) studied the dynamics of post-conflict debt. Their analysis emphasized the need for targeted debt relief, post-war reconstruction strategies, and international cooperation to address the unique challenges faced by countries emerging from conflict. These studies provide valuable insights into managing external public debt in the aftermath of uncertainty.

The unprecedented global shock caused by the COVID-19 pandemic has triggered a new wave of research on external public debt management under uncertainty. Napo (2022), Klutse et al. (2023) investigated the economic impact of the pandemic, emphasizing the need for adaptive fiscal policies and debt management strategies to mitigate the effects of the pandemic.

Oliynyk et al. (2017) explore optimal control models, that can provide valuable perspectives on decision-making processes in financial contexts. This knowledge can be beneficial for the forecasting article in terms of exploring alternative modelling approaches or refining existing methodologies for forecasting Ukraine's external public debt.

Forecasting external public debt is complex, involving integrating various economic variables and accounting for uncertainty.

One of the key aspects of the study is the analysis of existing methods for forecasting external public debt. As correctly noted by Husain et al. (2023), in general, there are three groups of methods for predicting socio-economic indicators: expert (intuitive), formalized (statistical), and combined. The advantage of expert (intuitive) methods is that they are used when quantitative information about the forecasting object is limited or absent. Formalized methods (methods of trend extrapolation; methods of correlation and regression analysis; methods of mathematical modelling) are based on logical analysis of facts, statistics, and forecast estimates using mathematical models.

Saidane (2019) and Lagoa et al. (2022) provided a comprehensive review of forecasting methods, discussing the advantages and limitations of time series models, econometric approaches, and scenario-based analysis. Their studies serve as a fundamental source for understanding the nuances of forecasting in the context of external public debt.

Accounting for uncertainty in forecasting models has been the focus of researchers’ attention. In particular, Mo and Kim (2016), Mersmann and Westermann (2020) have explored the role of uncertainty in debt sustainability analysis, emphasizing the need for probabilistic approaches to account for the dynamic nature of economic variables. This view of uncertainty links general forecasting methods and the challenges posed by external shocks.

Ademmer and Boysen-Hogrefe (2022) investigated the impact of forecasting errors on fiscal planning and debt accumulation. The study provides insights into the challenges of forecasting inaccuracies for debt management.

In addition, innovative approaches’ role in studying external public debt should be noted. For example, in recent years, machine learning and artificial intelligence methods have become famous for analyzing and forecasting economic processes. The works of scholars such as Andrada-Félix et al. (2015) explore the possibilities of applying these technologies in the context of forecasting external public debt. The authors provide examples of successful applications but also point out the challenges and limitations of such approaches.

Autoregressive models represent an essential category of forecasting methods that consider time dependencies in economic time series. Zhuravka et al. (2021a) and Caruk (2007) studied various methods and methodologies for forecasting public debt based on autoregressive models. Despite their power, these models need to be adapted to address the unique challenges of uncertainty, especially in the context of external public debt management.

Gnegne and Jawadi (2013) contributed to the literature by estimating the boundedness and nonlinearity of public debt dynamics using a threshold autoregressive (TAR) approach. This study adds a valuable contribution to understanding nonlinear patterns and thresholds in public debt dynamics.
So most of the scientific articles on public debt forecasting are based on constructing models of the interdependence of public debt with indicators of economic development. Indeed, these models are effective tools for analyzing and predicting possible trends in the growth or reduction of public debt.

However, under conditions of high uncertainty, which include factors such as military conflicts and the COVID-19 pandemic, it is necessary to look for more rational methods of forecasting public debt, which will allow for operational decisions. That is why it is important to study modern methods based on the analysis and forecasting of time series data, in which the projected values of external public debt will be determined by their values, but with a delay of a certain number of observations in the past.

AIMS AND OBJECTIVES

This article aims to analyze and apply existing time-series forecasting methodologies to determine the future values of Ukraine’s external public debt under uncertainty caused by the still unresolved consequences of the COVID-19 pandemic and Russia's full-scale war against Ukraine.

The article’s objectives are as follows:

▪ to identify methods of forecasting external public debt based on previous data;
▪ to investigate how each model accounts for uncertainty and its limitations in adapting to complex and unforeseen circumstances;
▪ to provide a detailed comparison of the forecasted values of external public debt obtained from each of the three forecasting methods;
▪ to explore in detail the risks associated with the limitations of these methods based on the need to ensure Ukraine's external debt security during wartime.

METHODS

The paper analyzes the annual and monthly data of Ukraine’s external public debt from December 31, 2011, to October 31, 2023.

Figure 1 shows the key methods, used in this study, for forecasting external public debt based on its previous values used in this study.

On the basis of the characteristics of the models shown in Figure 1, it is worth mentioning that the dynamics of Ukraine’s external public debt do not show seasonal dependence, so the use of seasonally adjusted models is unnecessary. Moreover, such an approach does not make a meaningful contribution to understanding the dynamics of debt indicators, but can also complicate the analysis without any additional benefit. In particular, in terms of forecasting, it is important to take into account that the absence of a seasonal component in the dynamics of external public debt simplifies the modelling of future trends.

\[
EPD = \alpha_0 t^n + \alpha_{n-1} t^{n-1} + \cdots + \alpha_2 t^2 + \alpha_1 t + \alpha_0
\]  

(1)

where, \(EPD\) – is the dependent variable (external public debt); \(t\) – independent variable (time); \(n\) – the degree of the polynomial (determined using the in-built forecasting function in Excel); \(\alpha_n, \alpha_{n-1}, \ldots, \alpha_0\) – are coefficients to be selected or calculated using approximation or regression methods. (determined using the built-in forecasting function in Excel).
Holt model - used to forecast time series where there is a trend or change in the average value over time. It includes a level (linear trend), a trend (change in the trend over time), and smoothing parameters for predicting future values.

Holt-Winters model - extends the Holt model by adding a seasonality component. This model is useful for forecasting in time series with seasonal changes or repeated cyclic fluctuations.

Autoregressive (AR) models are based on the idea that the current value of a time series depends on its previous values. The AR(p) model takes into account p previous values.

Moving average (MA) models describe the dependence of the current value on previous random errors of the model. The MA(q) model takes into account q previous errors.

The autoregressive moving-average model (ARMA) stands as a mathematical model employed for the examination and forecasting of stationary time series within the field of statistics. This model serves as a generalization of two less complex time series models, namely the autoregressive (AR) model and the moving average (MA) model.

ARIMA (Autoregressive Integrated Moving Average) - the model combines AR and MA, adding an integration component that reflects the degree of differentiation of the time series to ensure stationarity.

SARIMA (Seasonal ARIMA) is an extension of ARIMA to account for seasonality in time series.

Simple trend – the model takes into account a linear or non-linear trend in the time series without taking into account seasonality or other variations.

Trend with seasonality – in addition to the trend, this model includes seasonal components that take into account recurring changes in data in certain periods.

Exponential Smoothing (ETS). It includes models such as ETS(AAA), where each letter represents a different component of the model (trend, seasonality, smoothing).

Autoregressive and moving average models

Trend models – models that use linear regression to approximate and forecast time series

Also, for forecasting Ukraine's external public debt, a variant of the Holt exponential smoothing method (exponential smoothing with trend) was applied, which allows for the trend in the time series.

The formula for exponential Holt smoothing includes two steps: smoothing the series (formula 2) and calculating the trend (formula 3) (Saidane (2019)):

\[ PEPD_t = a \cdot EPD_t + (1 + a) \cdot PEPD_{t-1} + T_{t-1} \] (2)

where \( PEPD_T \) – is the projected value of external public debt at time \( t \); \( EPD_t \) – is the actual value of the external public debt at time \( t \); \( PEPD_{t-1} \) – is the predicted value at the previous time; \( T_{t-1} \) – is the trend value at the previous time point; \( a \) – smoothing coefficient, where \( 0 < a < 1 \).

\[ T_t = \beta \cdot (PEPD_t - PEPD_{t-1}) + (1 + \beta) \cdot T_{t-1} \] (3)

where \( T_t \) – is the trend value at time \( t \); \( \beta \) – is the smoothing coefficient for the trend, where \( 0 < \beta < 1 \).

Forecasted value of the external public debt for the future period (formula 4):

\[ PEPD_{t+h} = PEPD_t + h \cdot T_t \] (4)

were, \( h \) – the number of periods in the future.
The algorithm for forecasting external public debt based on an autoregressive model will have the following sequence of steps:

- checking the data for signs of persistence;
- stabilization of the time series. If the time series is non-stationary, use differentiation to stabilize it. This usually involves taking the first or second difference between consecutive values. Note that if the series is stationary and does not require transformation, this implies building an ARMA \((p, q)\) model. If the series is differentiated – ARIMA \((p, d, q)\);
- determine the parameters of the ARMA \((p, q)\) or ARIMA \((p, d, q)\) model. Where: \(p\) is the number of lags for auto-regression, determined by the analysis of the autocorrelation function (ACF); \(d\) is the number of differentiations, determined by the analysis of stationarity; \(q\) is the number of lags for the moving average, determined by the analysis of the partial autocorrelation function (PACF);
- building an ARMA/ARIMA model;
- use the resulting model to forecast future values.

The in-built data analysis tools in Excel and the EViews software package were used to build the models, plot the correlogram (graph), perform the Dickey-Fuller test, and calculate the model parameters.

**RESULTS**

The growth of external public debt under conditions of uncertainty requires new and improved existing mechanisms for its management, which, in the context of the changing economic environment, becomes critical to ensure the sustainability and efficiency of the country's financial system. Effective financial management during wartime requires promptness and flexibility in adjusting budget strategies to consider unforeseen circumstances.

When other factors cannot be considered due to uncertainty, it is essential to focus on adapting existing financial management methods based on available data.

For example, modern time-series analysis methods for forecasting external public debt can be an extremely effective tool. These methods are notable for their ability to predict future performance based on past data. They allow for seasonality, trends, and unexpected changes, making them ideal for situations where other factors may remain uncertain. Thus, modern time series analysis methods are becoming a key tool for dynamic and adaptive management of financial resources in a changing environment.

In addition to the models shown in Figure 1, the following methodologies are becoming increasingly popular:

- neural networks – for data forecasting, which can be used for complex and nonlinear time series forecasting models;
- combined models – combinations of different models to obtain better forecast accuracy;
- state space models – space and state models that can account for uncertainty and complex dynamic interactions in time series;
- scenario analysis – distribution of possible scenarios and assessment of the probability of different outcomes. Forecasting in this context is about getting an idea of what are the possible scenarios.

In general, it can be concluded that such a diverse range of methodologies and perspectives demonstrates the continuous development of the field of external public debt forecasting research, as well as the importance of a coherent and integrated approach to obtain the most accurate and reliable results.

Therefore, this study aims to determine the predicted values of Ukraine's external public debt until 31.12.2029 in order to identify the most appropriate method for forecasting external debt based on its previous values.

Extrapolating data based on a trend line can be one way to forecast external public debt. This method is one of the easiest to apply and can be effective for short-term or generalized forecasts. The simplicity of the method allows it to be used quickly and efficiently to get a general idea of the possible development of the situation with the country's external debt burden.

The results of forecasting Ukraine's external public debt based on the trend line are shown in Figure 2.
The external public debt forecasting results shown in Figure 2 represent a trajectory of debt movement, without specifying the values of its volumes.

After all, extrapolation based on the trend line assumes that the existing trend in external public debt will continue in the future. This assumption is based on the fact that previous trends in expenditures and debt may indicate that a certain pattern of change will continue in the future.

A polynomial trend line of the 2nd degree was chosen to extrapolate the external public debt data. In our opinion, this type of trend line provides the most accurate results, as it reflects the nature of the external debt dynamics and allows the main changes in its growth or decline rates to be preserved.

Thus, evident from the depicted graph, that the projected value of the external public debt will cross the threshold of USD 160,000 million.

This method is simple and affordable, especially when an analyst or researcher wants to get a quick and generalized forecast without spending much time and resources on complex models.

However, it is essential to keep in mind that in times of economic crises, political instability, or other crises where sudden and significant changes may occur, the trendline method may be ineffective and inadequate. In particular, the method may underestimate or oversimplify the situation in the case of complex and unforeseen changes. In general, it is important to recognize the limitations of the trend line method and to use it in the context of the specific conditions and circumstances of the external public debt time series under analysis.

Thus, this method can be used under conditions of uncertainty only if it is necessary to make short-term forecasts. This forecast is also convenient for inclusion in reports and presentation of results to stakeholders.

Another method that can be used to forecast external public debt is the exponential smoothing method.

This method uses weighted values of the external public debt, giving more weight to the most recent values.

The results of forecasting Ukraine’s external public debt using the exponential smoothing method are shown in Figure 3. The value of the smoothing coefficients is determined by the formula for the mean square error (MSE) and is equal to 0.21.
Figure 3 also shows the projected growth of Ukraine’s external public debt. However, unlike the previous results, it has an upward trend. In particular, the projected value of the external public debt – as of the end of 2029 – will amount to USD 127,781.51 million.

The exponential smoothing method of forecasting external public debt is attractive in conditions of uncertainty due to its ease of use and the possibility of quick forecasting. The small number of parameters in the method, such as smoothing coefficients, makes it less sensitive to poorly structured data and facilitates its use in conditions of economic uncertainty.

Also, the method may not be flexible enough to account for complex and unexpected changes in external public debt that may arise in times of uncertainty and economic instability.

Among the methods shown in Figure 1, one of the most effective is based on autoregressive and moving average models, where the dependent variable is determined by its own value, but with a delay of a certain number of observations in the past.

When creating autoregressive models for analyzing time series, it is important to take into account the peculiarities of their dynamics. For example, it is necessary to determine whether changes in the external public debt are random or have a «memory». The term «persistence» refers to the ability of a state to exist over a long period of time that extends beyond the time interval in which the state arose. The presence of persistence indicates the non-random nature of the data and indicates the possibility of using autoregressive models to analyze them.

In order to meet the needs of universality and to take into account the complexity of time series analysis, it is proposed to use the Hurst index as a measure of series persistence. This metric declines as the time lag between two identical pairs of values in the time series grows. The term «Hurst exponent» or «Hurst coefficient» was named in honour of G. E. Hurst, a leading researcher in this field (the standard symbol for the indicator is H).

To calculate the exponent, we used the R/S-analysis method as a method of normalized accumulated time series variance. The formula for calculating the Hearst exponent is as follows (5) (Zhuravka et al. (2019)):

\[
\log \left( \frac{R}{S} \right) = \log (c) + H \log (n)
\]

The segment that intersects the coordinate axis is defined as an estimate of the constant \( \log(c) \). The slope of the line in the regression equation serves as an estimate for Hurst’s H. Note that Hurst’s H can be within the interval \([0;1]\).

In the process of calculations, we obtained the following data series for the Logn and logRS indicators, which we need to build a regression equation (Table 1).

For the autoregressive model, we used monthly data of Ukraine’s external public debt from December 31, 2011, to October 31, 2023, in USD millions.

<table>
<thead>
<tr>
<th>Logn</th>
<th>logRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.383699</td>
</tr>
<tr>
<td>1.069181</td>
<td>0.43063</td>
</tr>
<tr>
<td>1.166091</td>
<td>0.60988</td>
</tr>
<tr>
<td>1.244263</td>
<td>0.644164</td>
</tr>
<tr>
<td>1.30103</td>
<td>0.613996</td>
</tr>
<tr>
<td>1.366121</td>
<td>0.660666</td>
</tr>
<tr>
<td>1.446303</td>
<td>0.84103</td>
</tr>
<tr>
<td>1.643213</td>
<td>0.863608</td>
</tr>
<tr>
<td>1.668141</td>
<td>0.9193</td>
</tr>
<tr>
<td>1.963233</td>
<td>1.126666</td>
</tr>
</tbody>
</table>

Based on the data in Table 1, we constructed the following regression equation (Figure 4).
Thus, the slope of the regression equation line, which is 0.747, indicates the value of the Hurst H. The high level of persistence indicates the presence of autocorrelation in the dynamics of external public debt. It confirms the possibility of using autoregressive models to analyze and forecast the volume of Ukraine’s external public debt.

The next step in building an autoregressive model is to check the series for stationarity.

The effectiveness of autoregressive forecasting methods relies significantly on the presence of stationarity. A time series is deemed stationary when its statistical characteristics, such as mean and variance, remain consistent over time, maintaining a constant distribution and unchanged autocorrelation.

In order to test whether a given time series is non-stationary, a variety of statistical methods and graphical tools can be used. One common approach is using a statistical stationarity test, such as the Dickey-Fuller test.

To ensure that this series of external public debt is stationary, the Dickey-Fuller test was applied, which allows for testing the hypothesis of a unit root in the time series. If the p-value is less than a specified threshold (usually 0.05), the hypothesis of non-stationarity is rejected and the series is considered stationary.

Also, the key indicators of the Dickey-Fuller test are the estimated and critical values of the τ-statistic.

The results of the test for analyzing and forecasting external public debt are shown in Table 2. It should be noted that the calculations were based on the use of the specialized software package Eviews.

<table>
<thead>
<tr>
<th>Augmented Dickey-Fuller test statistic</th>
<th>t-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test critical values: 1% level</td>
<td>-4.436071</td>
<td>0.0000</td>
</tr>
<tr>
<td>5% level</td>
<td>-1.478524</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-1.096457</td>
<td></td>
</tr>
</tbody>
</table>

The results of the test shown in Figure 5 are in favour of the stationarity of the series since the calculated value of the McKinnon test statistic is less than the critical value for the corresponding significance levels (with a minimum probability of error of 0% (since the p-value is zero)).

The stationarity check of the analyzed data showed their fundamental acceptability for use in autoregressive models.

Since the series of initial data of external public debt is stationary and does not require further transformations, the autoregressive moving-average model, ARMA, will be used to forecast its values. The ARMA model is an extension of two more straightforward time series models: the autoregressive model (AR) and the moving average model (MA).
To initially establish the overall structure of the future ARMA model and identify the lag count for each component, we will utilize the autocorrelation and partial autocorrelation function plots for the volume of external public debt.

Table 3 displays the calculated values for the autocorrelation function (ACF) and partial autocorrelation function (PACF) of Ukraine's external public debt series. The data in the table are aimed at making an initial assessment of the process type and determining the appropriate number of lags for inclusion in the ARMA model. The analysis of the table indicates significance for lag 1 (since the values of both functions are close to 1).

Table 3. Correlogram of the external debt of Ukraine.

<table>
<thead>
<tr>
<th>№</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>190.50</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.976</td>
<td>-0.016</td>
<td>377.41</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.965</td>
<td>0.007</td>
<td>560.88</td>
<td>0.000</td>
</tr>
<tr>
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<td>-0.041</td>
<td>740.61</td>
<td>0.000</td>
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<td>-0.036</td>
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<td>-0.019</td>
<td>1088.1</td>
<td>0.000</td>
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<td>1256.0</td>
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<td>-0.003</td>
<td>1420.2</td>
<td>0.000</td>
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<tr>
<td>9</td>
<td>0.887</td>
<td>-0.028</td>
<td>1580.5</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.873</td>
<td>-0.079</td>
<td>1736.4</td>
<td>0.000</td>
</tr>
<tr>
<td>11</td>
<td>0.858</td>
<td>0.002</td>
<td>1887.9</td>
<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>0.843</td>
<td>-0.047</td>
<td>2034.8</td>
<td>0.000</td>
</tr>
<tr>
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<td>2177.0</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.810</td>
<td>-0.004</td>
<td>2314.4</td>
<td>0.000</td>
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<tr>
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To generate predictive values for the modified time series of Ukraine's external public debt, we calculated parameter estimates for the ARMA model with varying numbers of parameters \((p, q)\) ranging from 1 to 12. Subsequently, models with the lowest values of SIC and AIC were chosen for further examination. The step-by-step estimation of the autoregressive (AR) components revealed that the ARMA \((1;5)\) specification demonstrated the minimum values of these criteria.

Hence, we believe that the ARMA \((1,5)\) model most precisely captures the pattern of external public debt dynamics, offering superior accuracy for subsequent forecasts. Equation (6) outlines the model specification in the EViews format:

\[
PEPD = 28459 + 0.8796 \times AR_{t-1} + 0.8745MA_{t-5}
\]  

(6)

The values of the coefficients \((0.8796\) and \(0.8745\)) indicate the strength of the relationship between the values of the series and their lags. These coefficients can help in understanding the dynamics and dependence between different periods of the time series.

The regression equation within the developed ARMA model is statistically significant, as evidenced by the Fisher's test values, and the residuals of the estimated final equation are white noise.

The forecast values of external public debt calculated on the basis of the ARMA model are shown in Figure 5.

As can be seen from Figure 5, the projected amount of external public debt is expected to increase to the level of 116,722.2 by the end of 2029.

However, it is worth noting that autoregressive models have several advantages, one of which is simplicity and clarity of interpretation. This makes it an effective tool for analyzing and forecasting external public debt, especially in resource-limited environments. Another advantage is that the ARMA model can identify certain trends and cyclical changes in time series, which helps to better understand long-term trends in public debt formation.

However, among the key drawbacks of using this model are the limited ability to take into account complex dependencies (the ARMA model assumes linear and stationary dependencies, so it may be limited in modelling complex and non-stationary characteristics of public debt) and sensitivity to outliers (ARMA may be sensitive to outliers or anomalies in the data. If the external public debt is subject to sudden and significant changes, which is not an exceptional phenomenon in times of war, this may affect the accuracy of forecasting.

Thus, based on the obtained forecast values of Ukraine's external public debt and the analysis of the advantages and disadvantages of each of these methods, it should be borne in mind that when using any of the time series analysis methods, the forecasts will be less accurate, since they are based mainly on historical data. Their effectiveness can be seen in short-term forecasting. It is also important to systematically evaluate the accuracy of forecasts and, if necessary, adapt models or consider other forecasting methods when new data becomes available or conditions change.
DISCUSSION

The use of extrapolation, which is based on identifying and continuing trends in historical data, is consistent with traditional time series forecasting methods. This approach has similarities with Ghosh (2015) [10], which emphasized the importance of recognizing patterns in economic variables for accurate forecasting. However, the effectiveness of trend-based extrapolation depends on the assumption that historical trends will continue, and this assumption can be questioned in the face of uncertainty caused by external shocks.

The use of exponential smoothing in this study reflects the recognition of the importance of recent observations in predicting future trends. This is in line with the findings of Caruk (2007) and Korneyev et al. (2022), who emphasized the need for adaptive forecasting methods that take into account changing economic conditions.

The inclusion of ARMA models in our forecasting methodology draws inspiration from seminal studies: Mersmann and Westermann (2020). ARMA models capture time dependencies in the data, offering a dynamic framework for forecasting. This method is well-known in the literature and provides a reliable basis for forecasting future values. However, ARMA models are not immune to challenges, especially in situations where external shocks cause volatility and disrupt the time patterns observed in historical data.

Comparing the findings with the existing literature, it becomes clear that each forecasting method has its own unique advantages and limitations. Extrapolation methods offer simplicity and transparency but can be unstable in the face of sudden changes. Exponential smoothing, while adaptive, requires careful adjustment of the parameters to match the degree of uncertainty. ARMA models, being autoregressive, reflect historical dependencies well, but can have problems in periods of high uncertainty when there are sharp deviations from historical patterns.

Our study contributes to the scientific discourse by integrating these three methods, and recognizing their strengths and risks. The combination of trend analysis, adaptive smoothing, and time dependence using ARMA models provides a better understanding of how external public debt can evolve under uncertainty. However, we recognize that no single forecasting method is universally preferable, and the choice of methodology should be context-specific, taking into account the nature and degree of uncertainty.

CONCLUSIONS

Uncertainty can complicate the forecasting of external public debt, and in such cases, some models based on several interdependent variables may not be effective enough, especially when it comes to making operational decisions. Therefore, this paper investigates methods of forecasting Ukraine's external public debt based on previous data.

The study's findings highlight the effectiveness of trend extrapolation, exponential smoothing, and autoregressive models in forecasting Ukraine's external public debt based on historical data, particularly for short-term projections and urgent decision-making. However, for the long-term management of the country's debt security, a more in-depth exploration of these methods is warranted, along with strategic conclusions drawn from integrating these models with indicators reflecting the country's economic development in the wartime context. Additionally, strategies should be developed to adapt to unforeseen changes, and models should be regularly updated to accurately capture current conditions.

The calculations unequivocally demonstrate an anticipated increase in Ukraine's external public debt amid the ongoing military conflict. Unchecked debt growth in uncertain conditions poses the risk of heightened financial strain on the country, escalating debt service costs, and constraining the implementation of crucial social and economic programs essential during wartime. In tandem with employing the forecasting methods outlined in this study, it is advisable to establish mechanisms for monitoring debt dynamics, analyzing risk factors, and promptly implementing corrective measures to mitigate adverse impacts. Furthermore, the development of adaptive forecasting scenarios, considering various levels of uncertainty and conflict dynamics, is essential for more effective financial planning and responsiveness to evolving economic conditions.

ADDITIONAL INFORMATION

AUTHOR CONTRIBUTIONS

Conceptualization: Mila Razinkova, Hanna Filatova
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Formal Analysis: Andriy Pylypenko, Oleksandr Firsov
Methodology: Mila Razinkova, Natalia Nebaba
Software: Oleksandr Firsov
Resources: Andriy Polypenko
Supervision: Andriy Polypenko
Validation: Hanna Filatova, Natalia Nebaba, Fedir Zhuravka
Investigation: Mila Razinkova, Andriy Polypenko, Natalia Nebaba, Oleksandr Firsov, Fedir Zhuravka
Visualization: Mila Razinkova, Hanna Filatova, Natalia Nebaba
Project administration: Mila Razinkova, Fedir Zhuravka
Writing – review & editing: Mila Razinkova, Hanna Filatova
Writing – original draft: Mila Razinkova, Hanna Filatova

REFERENCES


Разінькова М., Філатова Г., Пилипенко А., Небаба Н., Фірсов О., Журавка Ф.  

**ПРОГНОЗУВАННЯ ЗОВНІШНЬОГО ДЕРЖАВНОГО БОРГУ УКРАЇНИ В УМОВАХ НЕВИЗНАЧЕНОСТІ**

Прогнозування зовнішнього державного боргу в умовах невизначеності важливе, оскільки дозволяє країні адекватно реагувати на економічні та фінансові вигадки, сприяє ефективному управлінню фінансовими ресурсами, формуванню стабільної фінансової політики та забезпеченню зовнішньої боргової безпеки країни, що є критичними елементами для забезпечення економічної стійкості й сталого розвитку. Основною метою дослідження є критичний
аналіз і застосування існуючих методологій прогнозування часових рядів для визначення майбутніх значень зовнішнього державного боргу України в умовах невизначеності, що зумовлена досі невирішеними наслідками пандемії COVID-19, вторгненням та активною війною росії в Україні. За допомогою трьох методів прогнозування, а саме екстраполяції на основі лінії тренду, експоненціального згладжування та авторегресійної моделі, спрогнозовано обсяги зовнішнього державного боргу до 2029 року та представлено графічне зображення динаміки боргу з 2011 по 2029 рік. Найбільш пессимістичний прогноз щодо зростання зовнішнього державного боргу виявлений при застосуванні методу екстраполяції даних на основі лінії тренду. Порівняльний аналіз прогнозних значень за трьома методами прогнозування дозволив виявити спільні тенденції щодо зростання державного боргу, а також визначити ключові переваги та недоліки, притаманні кожній моделі. Важливо, що в статті наголошено на спільні ризики, виявлених при прогнозуванні зовнішнього боргу України, при використанні моделей аналізу часових рядів, включаючи проблему досягнення лише короткострокової точності прогнозування та недостатньої гнучкості для врахування складних і неочікуваних змін, що може виникнути в умовах невизначеності та економічної нестабільності. Результати дослідження дають ціну інформацію для політиків та зацікавлених сторін, які намагаються зорієнтуватися в труднощах управління зовнішнім державним боргом в умовах невизначеності.

Ключові слова: зовнішній державний борг, прогнозування, невизначеність, управління державним боргом, економічний розвиток, Україна, воєнний час

JEL Класифікація: H60, H63, C53, F34