MODERN TOOLS FOR FORMING THE OPTIMAL STRUCTURE OF REVENUES OF TERRITORIAL COMMUNITIES' BUDGETS

ABSTRACT

The influence of budgetary risks on the formation of revenues of budgets of territorial communities is investigated and a methodical approach to estimating the structure of revenues of budgets of territorial communities in the conditions of financial decentralization is developed. It is noted that today the introduction of the latest methods for optimizing the revenues of the budgets of territorial communities with permanent control over the level of budgetary risks becomes especially important. Mathematical forecasting of the probability of budget risks and its empirical measurement requires special attention. The main approaches of domestic and foreign scientists to forecasting the probability of budget risks with the help of economic and mathematical tools are considered. Its practical use, on the one hand, will ensure their permanent control, and on the other - will allow optimization of the revenues of the budgets of territorial communities. It is established that the existing methods are difficult to implement in domestic practice due to their significant mathematical calculation. Calculations were made using the postulates of portfolio theory, in particular the portfolio approach of J. Hicks, which allowed to form a rational structure of revenues of budgets of territorial communities in terms of general and special funds, as well as to identify ways to minimize budgetary risks to ensure optimal revenue structure without interbudgetary transfers in conditions of permanent crisis phenomena. This will allow local governments of territorial communities to increase the performance of their budgets. The latest tools of financial decision-making using J. Hicks' portfolio approach to the formation of the optimal structure of budget revenues of territorial communities without intergovernmental transfers, taking into account the level of budgetary risk, were obtained by applying the tools of economic and mathematical modeling. It is concluded that the latest tools for the formation of the optimal structure of budget revenues of territorial communities with permanent forecasting of the maximum possible level of their growth under the control of budget risks allow the development of ways to minimize, forming many scenarios for the optimal structure of future budget revenues and budget risk. It is established that the proposed toolkit allows to generate alternative financial solutions for local governments and assess the results and consequences of their possible adoption in determining strategic guidelines for the development of local community finances. It is proposed to further use the developed scientific and methodological approach to optimize the revenue structure of budgets of territorial communities in the context of various budgetary risks, which, of course, will allow local governments to make effective financial decisions in developing programs of socio-economic development of territorial communities.

Keywords: financial capacity of the territorial community, finances of territorial communities, a budget of the territorial community, financial decentralization, financial system, budget risk, local borrowings, revenues of the budget of the territorial community

JEL Classification: H72, H77

INTRODUCTION

The realities of financial decentralization create prerogatives for the effective socio-economic development of territorial communities (TC), which is reflected in the indicators of their budgets. The reform of local self-government has created conditions for increasing the financial capacity and sustainability of local territorial communities through the
expansion of budget revenues. This led to their absolute growth, providing opportunities to adequately respond to budgetary risks. At the same time, adequate response to various budget risks requires real budget planning, which aims to optimize the structure of revenues and expenditures, increase own resources, and develop methodological and economic-mathematical tools, which will increase the financial capacity and self-sufficiency of territorial communities.

LITERATURE REVIEW

A critical study of the latest research and publications that have begun to address this issue is in the field of view of many scientists. In particular, the definition of budgetary risks at different levels of taxonomy is considered in the works of Khokhlov N. [8] and Khokhlova N. [9], however, they do not provide a mathematical apparatus for their identification or measurement. Their mathematical justification is considered by Raevsky K. and Eleiko J. [12], Eigel F. [6], Bulgakova S, Mykytyuk I. [3], Balatsky E. and Spitzgluz S. [2], J. Hicks [8]. The development of economic and mathematical tools for assessing budgetary risks is reflected in the works of Kushlyk-Divulksa I. and Kushlyk B. [10], Vitlinsky V. [14], however, the achievements of these scientists are not characterized by the development of tools for quantitative measurement of budget risks. Hrynchyshyn I. [7], Dub A. [4], Kravtsiv V., and Storonianska I. [13] in their research focus on finding out possible losses of local budgets due to various risks, but the issue of a quantitative assessment of each type of risk is not covered. Kutsenko T. and Verbytska A. [11] in scientific works review the basic tools that have already been created and used to model and assess the budget capacity of local communities, but scientists do not mention the impact of budget risks.

At the same time, the most common method of quantitative risk assessment of regional budgets, proposed and used by scientists, in particular, Ayupova S. [1] is a statistical analysis using variance, standard deviation, and coefficient of variation. These indicators are usually calculated for the budget execution indicator (the risk of non-fulfillment of the planned budget indicators is determined). In turn, according to Yumit Karagozlyu [15], for a full and objective assessment of local budget risks, there is a need to take into account a wider range of uncertainty characteristics in the field of budget planning and execution, in particular, risk planning and budget indicators the impact of risk factors. A thorough analysis of scientific papers shows that, despite the rather significant and thorough scientific and theoretical developments, research and development of economic and mathematical tools for assessing budgetary risks in general and territorial community budgets, in particular, have not yet been properly reflected. However, a thorough analysis of scientific papers on this subject shows that, despite the rather significant and thorough scientific and theoretical developments, research and development of economic and mathematical tools for assessing budgetary risks in general and territorial community budgets, in particular, have not yet been properly reflected in the works of modern scientists.

However, all further scientific results of this work will be based on the generalization of conclusions obtained by domestic and foreign scientists, in order to develop an appropriate methodological basis for studying the probability of occurrence and measuring the level of risk of local budgets based on their own economic and mathematical approach.

AIMS AND OBJECTIVES

The aim of the article is to develop a scientific and methodological approach using economic and mathematical modeling tools to assess the structure of budget revenues of territorial communities and form, on this basis, proposals for their optimization in the context of permanent control of budgetary risks. Achieving this goal requires solving the following tasks:

▪ highlight the theoretical postulates for the development of a methodological approach to optimize the structure of budget revenues of local communities, using portfolio analysis by J. Hicks;

▪ develop stages of economic and mathematical modeling of the optimal structure of budget revenues of territorial communities at a given (controlled) level of budget risk;

▪ model and visualize the economic-mathematical model of the optimal structure of budget revenues of territorial communities by selected stages, using trends and results identified during the analysis of structural imbalances in their budget revenues and identification of budget risks.
METHODS

In order to achieve this goal and to solve the set task, we used general and special scientific and economic-mathematical methods. In particular, empirical, mathematical, and statistical methods were used in the research and analytical assessment of the structure of revenues of budgets of territorial communities: descriptive-statistical, structural-index methods - to find out the determinants of imbalances in revenues of budgets of territorial communities; comparative analysis - in order to identify imbalances in the structure of budget revenues of territorial communities; directions for optimizing the structure of budget revenues of territorial communities at a controlled level of budget risk were developed using correlation-regression modeling, formalization, and tools of economic modeling.

Directly in the development of economic and mathematical models of such a plan, the foreign practice uses statistical methods of risk assessment, the essence of which is to determine the probability of losses based on statistics from the previous period and establish the area (zone) of risk, risk factor and more. It is also possible to use the method of simulation, which is one of the effective methods of analysis of the economic system. In turn, the technology "Risk Metrics", developed by J. P. Morgan can also be used to assess budgetary risk, although it is adapted to the securities market. Using the method of reliable equivalents, the expected values of the flow of payments are adjusted by introducing special reduction factors in order to bring the expected revenues to the values of payments. The calculation of these coefficients is practically unquestionable, and their values can be reliably determined, which is relevant for budget revenues and expenditures. Taking into account the main methods of risk assessment used in foreign countries, as well as based on our own developed mathematical apparatus, we predicted the probability of occurrence and empirically studied the budgetary risks of local communities, and assessed the revenue structure of local communities.

RESULTS

In practice, the finances of territorial communities as a socio-economic system are characterized by stages of functioning and development. In the area of functioning, the finances of territorial communities as a system are supported in the set parameters, and the functions that determine the integrity of the system are preserved. At the same time, development is a dynamic process that forces the finances of territorial communities as a system to improve their socio-economic parameters and increase financial performance. Accordingly, it can be stated that ensuring the strategic development of TC finances is associated with their functioning in the context of the established TC budgets with the optimal revenue structure at a controlled level of budget risk. The implementation of such an approach is possible under the condition of making strategic financial decisions based on advanced methods of their modeling. We modeled the vectors of strategic development of TC finances, using trends and results identified during the analysis of structural imbalances in TC budget revenues and identification of budget risks.

We consider acceptable in the context of our study the work of J. Hicks [8], in particular using the postulates of portfolio theory. At the same time, we are convinced that the problem of measurability of goals, values, and criteria is extremely important for the implementation of the procedure for choosing the optimal financial solution [9]. Making the best financial decision involves achieving specific goals, which are used to compare alternative financial decisions with each other in the level of preference One solution is better than another only if it is better suited to the intended purpose. The mathematical expression or model of the goal is the target function (criterion) of the quality of the financial decision, which in the case of optimization is determined by the criterion of optimality.

Accordingly, the vector of development of finances of territorial communities lies in the area of the need to form the optimal structure of TC budget revenues (excluding intergovernmental transfers) by the criterion of maximizing the level of TC budget revenues growth at a given level of budget risk.

Thus, we model a situation in which it is necessary to determine the following proportions of the distribution of budget revenues of the territorial community without taking into account intergovernmental transfers (tax revenues , non-tax revenues – income from capital transactions – , where , the share of budget revenues of the territorial community excluding inter-budget transfers, which fall on the -th type of revenues), so that the budget risk on budget revenues of the territorial community , so that the budget risk on budget revenues of the territorial community at a given expected level of revenue growth of the territorial community without inter-budget transfers was set (controlled). We model the problem of forming the optimal structure of TC budget revenues without taking into account intergovernmental transfers in a slightly different formulation: at a given level of budget risk (variation) of TC budget revenues without taking into account intergovernmental transfers , we have to find such shares of revenues that can maximize the level of TC budget revenues without taking into account intergovernmental transfers.
\[
\max_{\mu_1, \mu_2, \mu_3} \mu_d = \max(\mu_1 y_1 + \mu_2 y_2 + \mu_3 y_3)
\]  
(1)

In the mathematical formulation of the developed financial problem we have:
\[
\sigma_1^2 y_1^2 + \sigma_2^2 y_2^2 + \sigma_3^2 y_3^2 + 2r_{12} \sigma_1 \sigma_2 y_1 y_2 + 2r_{13} \sigma_1 \sigma_3 y_1 y_3 + 2r_{23} \sigma_2 \sigma_3 y_2 y_3 = D_d
\]  
(2)

where \(\min(\sigma_1^2, \sigma_2^2, \sigma_3^2) < D_d < \max(\sigma_1^2, \sigma_2^2, \sigma_3^2)\).

To solve the financial problem, we substitute the formula \(y_1 = 1 - y_2 - y_3\) in equation (3), and obtain the following equality:
\[
D_d = \sigma_1^2 (1 - y_2 - y_3)^2 + \sigma_2^2 y_2^2 + \sigma_3^2 y_3^2 + 2r_{12} \sigma_1 \sigma_2 (1 - y_2 - y_3)y_2 + 2r_{13} \sigma_1 \sigma_3 (1 - y_2 - y_3)y_3 + 2r_{23} \sigma_2 \sigma_3 y_2 y_3
\]  
(3)

In order to find \(y_2\) we have to form the equation:
\[
(\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2)y_2^2(\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2)y_2^2 + 2[r_{23} \sigma_2 \sigma_3 y_3 - r_{13} \sigma_1 \sigma_3 y_3 + r_{12} \sigma_1 \sigma_2 (1 - y_3) - \sigma_1^2 (1 - y_3)]y_2 + \sigma_2^2 (1 - y_3)^2 + \sigma_3^2 y_3^2 + 2r_{13} \sigma_1 \sigma_3 (1 - y_3)y_3 - D_d = 0
\]  
(4)

The coefficient \(y_2^2\) in equation (4) is positive, and can be zero only in the case of equality of standard deviations of the first and second components of TC budget revenues (\(\sigma_1 = \sigma_2\)) without taking into account intergovernmental transfers and with their full correlation \(r_{12} = 1\): \(\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2 \geq \sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2 = (\sigma_1 - \sigma_2)^2 \geq 0\) (which is unlikely in practice).

Therefore, let the more general condition for the positiveness of the expression be satisfied:
\[
\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2 > 0
\]  
(5)

If this inequality holds, then the positive solution of equation (4) can be written as:
\[
y_2 = [\sigma_1^2 (1 - y_3) - r_{12} \sigma_1 \sigma_2 (1 - y_3) + r_{13} \sigma_1 \sigma_3 y_3 - r_{23} \sigma_2 \sigma_3 y_3 + ((\sigma_1^2 (1 - y_3) - r_{12} \sigma_1 \sigma_2 (1 - y_3) + r_{13} \sigma_1 \sigma_3 y_3 - r_{23} \sigma_2 \sigma_3 y_3)^2 - (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2)(\sigma_1^2 (1 - y_3)^2 + \sigma_2^2 y_2^2 + 2r_{13} \sigma_1 \sigma_3 (1 - y_3)y_3 - D_d))]^{1/2} / (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2).
\]  
(6)

Substituting formula (6) in (2), we write the expected level of growth of TC budget revenues without taking into account intergovernmental transfers as:
\[
\mu_d = \mu_1 + (\mu_3 - \mu_1) y_3 + (\mu_2 - \mu_1) \times [\sigma_1^2 (1 - y_3) - r_{12} \sigma_1 \sigma_2 (1 - y_3) + r_{13} \sigma_1 \sigma_3 y_3 - r_{23} \sigma_2 \sigma_3 y_3 + ((\sigma_1^2 (1 - y_3) - r_{12} \sigma_1 \sigma_2 (1 - y_3) + r_{13} \sigma_1 \sigma_3 y_3 - r_{23} \sigma_2 \sigma_3 y_3)^2 - (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2)(\sigma_1^2 (1 - y_3)^2 + \sigma_2^2 y_2^2 + 2r_{13} \sigma_1 \sigma_3 (1 - y_3)y_3 - D_d))]^{1/2} / (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2).
\]  
(7)

Find the differential of equation (7) for the variable \(y_3\):
\[
\frac{d \mu_d}{d y_3} = \frac{\mu_3 - \mu_1 + (\mu_2 - \mu_1) \times [r_{12} \sigma_1 \sigma_2 + r_{13} \sigma_1 \sigma_3 - r_{23} \sigma_2 \sigma_3 + \frac{1}{2} \times ((\sigma_1^2 (1 - y_3) - r_{12} \sigma_1 \sigma_2 (1 - y_3) + r_{13} \sigma_1 \sigma_3 y_3 - r_{23} \sigma_2 \sigma_3 y_3)^2 - (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2)(\sigma_1^2 (1 - y_3)^2 + \sigma_2^2 y_2^2 + 2r_{13} \sigma_1 \sigma_3 (1 - y_3)y_3 - D_d))]^{1/2} x (2(\sigma_1^2 (1 - y_3) - r_{12} \sigma_1 \sigma_2 (1 - y_3) + r_{13} \sigma_1 \sigma_3 y_3 - r_{23} \sigma_2 \sigma_3 y_3) \times (r_{12} \sigma_1 \sigma_2 + r_{13} \sigma_1 \sigma_3 - r_{23} \sigma_2 \sigma_3 - \sigma_1^2 - (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2) \times x (2(\sigma_2^2 y_2^2 - 2r_{12} \sigma_1 \sigma_2 (1 - y_3) + 2r_{13} \sigma_1 \sigma_3 - 4r_{13} \sigma_1 \sigma_3 y_3))) / (\sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2)
\]  
(8)

To simplify, we introduce the notation:
\[
a_1 = r_{12} \sigma_1 \sigma_2 + r_{13} \sigma_1 \sigma_3 - r_{23} \sigma_2 \sigma_3 - \sigma_1^2;
\]  
(9)
\[
a_2 = \sigma_1^2 + \sigma_2^2 - 2r_{12} \sigma_1 \sigma_2.
\]  
(10)

We equate the derivative to zero and get:
\[
\mu_3 - \mu_1 + (\mu_2 - \mu_1) \times [a_1 + \frac{1}{2}((\sigma_1^2 - r_11\sigma_1\sigma_2 + r_12\sigma_1\sigma_2 + r_13\sigma_1\sigma_3 - r_23\sigma_2\sigma_3 - \sigma_1^2)y_3)^2 - a_2(\sigma_1^2 - D_d + \\
+ 2(r_13\sigma_1\sigma_3 - \sigma_1^2)y_3 + (\sigma_1^2 + \sigma_2^2 - 2r_13\sigma_1\sigma_3)y_3^2))^{\frac{3}{2}} \times (2a_1(\sigma_1^2 - r_12\sigma_1\sigma_2 + a_3y_3) - 2a_2(\sigma_3^2y_3 - \sigma_1^2(1 - y_3) + \\
+ r_13\sigma_1\sigma_3 - 2r_13\sigma_1\sigma_3y_3))/a_2 = 0. 
\]  
(11)

To simplify the transformation, we introduce the notation:

\[
a_3 = a_1^2 - r_12\sigma_1\sigma_2, 
\]
(12)

\[
a_4 = r_13\sigma_1\sigma_3 - \sigma_1^2, 
\]
(13)

\[
a_5 = a_1^2 + \sigma_3^2 - 2r_13\sigma_1\sigma_3. 
\]
(14)

Taking into account expressions (12–14), to simplify the transformations we write equation (15) as:

\[
\mu_3 - \mu_1 + (\mu_2 - \mu_1) \times [a_1 + \frac{1}{2}((a_1 + a_1y_3)^2 - a_2(\sigma_1^2 - D_d + 2a_4y_3 + a_5y_3^2))^{\frac{3}{2}} \times (2a_1(a_1 + a_1y_3) - 2a_2(a_4 + \\
+ a_5y_3))/a_2 = 0. 
\]
(15)

After mathematical transformations we get:

\[
[(\mu_3 - \mu_1)a_2 + (\mu_2 - \mu_1)a_4] \times [a_3^2 - a_2(\sigma_1^2 - D_d) + 2(a_1a_3 - a_2a_4)y_3 + (a_5^2 - a_2a_5)y_3^2]^{\frac{1}{2}} = (\mu_1 - \mu_2)[a_1a_3 - \\
a_2a_4 + + (a_1^2 - a_2a_2)y_3]. 
\]
(16)

For simplicity, we denote:

\[
b_1 = a_1a_3 - a_2a_4, 
\]
(17)

\[
b_2 = a_1^2 - a_2a_5. 
\]
(18)

Substituting the notation in the formula (16), we obtain:

\[
(Kb_2 - (\mu_1 - \mu_2)^2b_2^2)y_3^2 + 2(Kb_1 - b_1b_2(\mu_1 - \mu_2)^2)y_3 + K(a_3^2 - a_2(\sigma_1^2 - D_d)) - b_1^2(\mu_1 - \mu_2)^2 = 0, 
\]
(19)

where \(K = [(\mu_3 - \mu_1)a_2 + (\mu_2 - \mu_1)a_4]^2. 
\)
(20)

If \(Kb_2 - (\mu_1 - \mu_2)^2b_2^2 > 0, 
\)
(21)

then the positive solution of equation (19) will look like:

\[
y_3 = \frac{b_1b_2(\mu_1 - \mu_2)^2 - Kb_1 - ((b_1b_2(\mu_1 - \mu_2)^2 - Kb_1)^2 - (Kb_2 - (\mu_1 - \mu_2)^2b_2^2) \cdot (K(a_3^2 - a_2(\sigma_1^2 - D_d)) - \\
- b_1^2(\mu_1 - \mu_2)^2))^\frac{1}{2}}{((Kb_2 - (\mu_1 - \mu_2)^2b_2^2). 
\]
(22)

Substituting (22) into (16), we obtain the optimal share \(y_3. \) If the inequality opposite to (21) holds, then:

\[
y_3 = \frac{b_1b_2(\mu_1 - \mu_2)^2 - Kb_1 - ((b_1b_2(\mu_1 - \mu_2)^2 - Kb_1)^2 - (Kb_2 - (\mu_1 - \mu_2)^2b_2^2) \times (K(a_3^2 - a_2(\sigma_1^2 - D_d)) - \\
- b_1^2(\mu_1 - \mu_2)^2))^\frac{1}{2}}{((Kb_2 - (\mu_1 - \mu_2)^2b_2^2). 
\]
(23)

Consider the partial case of problem (12), (13), in which the components of TC budget revenues without taking into account intergovernmental transfers are independent of each other:

\[
r_{23} = r_{13} = r_{23} = 0. 
\]
(24)
Then equality (14) is simplified:

\[(\sigma_1^2 + \sigma_2^2)y_2^2 - 2\sigma_1^2(1 - y_3)y_2 + \sigma_1^2(1 - y_3)^2 + \sigma_2^2y_3^2 - D_d = 0.\]  \hfill (25)

Accordingly, the solution (25) relatively \(y_2\) will look:

\[y_2 = \frac{(\sigma_1^2(1 - y_3) + \sqrt{(D_d - \sigma_1^2y_3^2)(\sigma_1^2 + \sigma_2^2) - \sigma_2^2y_2^2(1 - y_3)^2})}{(\sigma_1^2 + \sigma_2^2)}\]  \hfill (26)

Formula (26) for the expected level of growth of TC budget revenues without taking into account intergovernmental transfers also becomes simpler:

\[\mu_d = \mu_1 + (\mu_3 - \mu_1)y_3 + (\mu_2 - \mu_1) \times \left(\frac{(D_d - \sigma_1^2y_3^2)(\sigma_1^2 + \sigma_2^2) - \sigma_2^2y_2^2(1 - y_3)^2}{(\sigma_1^2 + \sigma_2^2)} + \sigma_1^2(1 - y_3)\right).\]  \hfill (27)

Therefore, we examine the range of allowable values of the expected level of revenue growth of the TC budget without taking into account intergovernmental transfers by formula (27), solving this inequality:

\[(D_d - \sigma_2^2y_2^2)(\sigma_1^2 + \sigma_2^2) - \sigma_1^2\sigma_2^2(1 - y_3)^2 \geq 0.\]  \hfill (28)

Expression (28) can be written as follows:

\[z = \sigma_2^2(\sigma_1^2 + \sigma_2^2)y_2^2 + \sigma_1^2\sigma_2^2(1 - y_3)^2 - D_d(\sigma_1^2 + \sigma_2^2) \leq 0,\]  \hfill (29)

where \(0 \leq y_3 \leq 1\).

Calculations confirm that \(z\) as a function of \(y_3\) is convex because \(\frac{d^2z}{dy_3^2} = 2\sigma_2^2(\sigma_1^2 + \sigma_2^2) + 2\sigma_1^2\sigma_2^2 > 0\). The suitability of this derivative means precisely the convexity of the function \(z\), which indicates that the function can acquire its greatest value only at the ends of the interval where it is defined, when \(y_3 = 0\) or \(y_3 = 1\) \([4]\).

If, \(y_3 = 0\) we get:

\[z = \sigma_1^2\sigma_2^2 - D_d(\sigma_1^2 + \sigma_2^2),\]  \hfill (30)

If \(y_3 = 1\), we get:

\[z = (\sigma_2^2 - D_d)(\sigma_1^2 + \sigma_2^2).\]  \hfill (31)

Inequalities (29) and formulas (30) and (31) imply restrictions on the variation of TC budget revenues without taking into account intergovernmental transfers:

\[D_d \geq \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad \text{and} \quad D_d \geq \sigma_2^2.\]  \hfill (32)

The last inequality agrees with condition (13), if the variation of the third component of TC budget revenues is the smallest of the variations of all components of community budget revenues without taking into account intergovernmental transfers. And this can always be achieved by changing the numbering of TC budget revenue shares in the modeling process, which will allow the use of the developed tools, regardless of the order of numbering of the components of community budget revenues.

Assuming that conditions (32) are satisfied, we investigate function (27) to the maximum. To do this, we calculate its derivative by the variable \(y_3\):

\[\frac{d\mu_d}{dy_3} = (\mu_3 - \mu_1) + (\mu_2 - \mu_1) \times \left(D_d - \sigma_2^2y_3^2\right)(\sigma_1^2 + \sigma_2^2) - \sigma_1^2\sigma_2^2(1 - y_3)^2 \right)^{\frac{1}{2}} \times \frac{\sigma_1^2\sigma_2^2(1 - y_3)^2}{(\sigma_1^2 + \sigma_2^2)} - \frac{(\mu_3 - \mu_1)y_3^2}{(\sigma_1^2 + \sigma_2^2)}.\]  \hfill (33)
Function (33) is constant only if the growth levels of the first and second components of TC budget revenues excluding intergovernmental transfers are equal to each other: $\mu_1 = \mu_2$.

Therefore, we will assume that inequalities hold:

$$
(\mu_2 - \mu_1) > 0; \quad (\mu_3 - \mu_1) > 0; \quad (\mu_3 - \mu_2) > 0.
$$

(34)

Under condition (34) we equate the obtained derivative (33) to zero and get:

$$
\frac{(\mu_3 - \mu_1)(\sigma_1^2 + \sigma_2^2)((\mu_2 - \sigma_1^2)(\sigma_2^2 - \sigma_1^2) - \sigma_1^2 \sigma_2^2(1 - y_3))^2}{(\mu_1 - \mu_2)^2} = -\sigma_1^2(\sigma_2^2 + \sigma_3^2)y_3 + \sigma_1^2 \sigma_2^2(1 - y_3).
$$

(35)

The left-hand side of equation (35) acquires only positive values if

$$
K = \frac{\mu_3 - \mu_1}{\mu_1 - \mu_2} > 0, \quad M_3 = \mu_3 - \frac{\mu_3 - \mu_1}{\sigma_1^2 + \sigma_2^2} \sigma_1^2.
$$

(36)

Inequality (36) is fulfilled if the first component of TC budget revenues - tax revenues - has the lowest expected level of growth:

$$
\mu_1 = \min\{\mu_1, \mu_2, \mu_3\} \quad \mu_1 > M_3
$$

(37)

or the highest expected level of growth:

$$
\mu_1 = \max\{\mu_1, \mu_2, \mu_3\} \quad \mu_1 < M_3.
$$

(38)

If condition (36) is satisfied, then for the existence of a real solution of equation (35) it is necessary that the right-hand side of this equation is positive:

$$
\sigma_1^2 \sigma_2^2(1 - y_3) - \sigma_2^2(\sigma_2^2 + \sigma_3^2)y_3 > 0.
$$

(39)

The solution of inequality (39) will be as follows:

$$
y_3 < \frac{\sigma_2^2}{\sigma_1^2 \sigma_2^2 + \sigma_2^2 \sigma_3^2 + \sigma_1^2 \sigma_3^2}.
$$

(40)

Under conditions (36) and (40), equation (35) can be squared:

$$
\frac{(M_3 - \mu_1)^2(\sigma_1^2 + \sigma_2^2)^2(\sigma_1^2 - \sigma_3^2)(\sigma_2^2 - \sigma_3^2)(1 - y_3)^2}{(\mu_1 - \mu_2)^2} = (\sigma_1^2 \sigma_2^2 - (\sigma_1^2 \sigma_2^2 + \sigma_2^2 \sigma_3^2 + \sigma_1^2 \sigma_3^2)y_3)^2.
$$

(41)

To simplify the transformations, we convert equation (41) to the canonical form:

$$
\frac{(D_1 + D_2 + D_3)^2(\mu_2 - \mu_1)(y_3 - y_3)^2}{D_1 D_2 D_3} = (\sigma_1^2 \sigma_2^2 - (\sigma_1^2 \sigma_2^2 + \sigma_2^2 \sigma_3^2 + \sigma_1^2 \sigma_3^2)y_3)^2.
$$

(42)

where $D_i = \sigma_i^2, \quad (i = 1, 3)$.

Equation (42) will be linear if the condition is satisfied:

$$
K = \frac{M_3 - \mu_1}{\mu_1 - \mu_2} = \frac{\sqrt{D_1 D_2 D_3}}{D_1 + D_2}.
$$

(43)

If expression (43) holds, equation (42) will have a solution:

$$
y_3 = \frac{(M_3 - \mu_1)^2(\sigma_1^2 + \sigma_2^2 - (\sigma_1^2 + \sigma_2^2)(\sigma_1^2 + \sigma_2^2) D_3^2)}{2D_1 D_2 (D_1 + D_2) (M_3 - \mu_1)^2 (\sigma_1^2 + \sigma_2^2)(\sigma_1^2 + \sigma_2^2)}
$$

(44)
Taking into account condition (43), formula (44) can be transformed as follows:

$$y_3 = \frac{D_1^2(D_2 + D_1 D_3 + D_2 D_3 - D_3 D_2) (D_1 D_2 - D_1 D_3)}{4D_1^2(D_1 D_2 + D_2 D_3 + D_3 D_1)}.$$  \hspace{1cm} (45)

Substituting formula (45) into inequality (42), we obtain the following condition:

$$D_d > \frac{D_1 D_2 (D_1 D_2 + D_2 D_3 - 2D_1 D_3)}{(D_1 + D_2)(D_1 D_2 + D_2 D_3 + D_3 D_1)}.$$  \hspace{1cm} (46)

In addition to condition (46), it is necessary to find the condition of variation of TC budget revenues without taking into account intergovernmental transfers, under which the solution (45) will be positive:

$$D_d < \frac{D_1 D_2 (D_1 D_2 + D_2 D_3 - 2D_1 D_3)}{(D_1 + D_2)(D_1 D_2 + D_2 D_3 + D_3 D_1)}.$$  \hspace{1cm} (47)

Thus, under conditions (43), (46) and (47), the value calculated by formula (45) is the extreme point of growth of TC budget revenues without taking into account intergovernmental transfers. In order to determine whether the obtained condition determines the minimum or maximum level of their growth, we calculate the second-order derivative:

$$\frac{d^2y_3}{dy_3^2} = \frac{\mu_2 - \mu_1}{D_1 + D_2}(-D_1 D_2 + D_1 D_3 + D_2 D_3) \left( D_d - D_3 y_3^2 \right) (D_1 + D_2) - D_1 D_2 (1 - y_3^2)^2 \left( \frac{1}{2} \left[ (D_d - D_3 y_3^2) (D_1 + D_2) - D_1 D_2 (1 - y_3^2) \right] - 2D_1 (D_1 + D_2) y_3 + D_1 D_2 (1 - y_3) \right)^2.$$  \hspace{1cm} (48)

As confirmed by the obtained formula, the second derivative is negative when $\mu_2 - \mu_1 > 0$, the expected level of growth of the TC budget revenue component excluding intergovernmental transfers exceeds the same indicator of the first component of the community budget revenue. Therefore, formula (45) can be used to determine the share of the third component of TC budget revenues without taking into account intergovernmental transfers, if condition (47) is met. So, for example, if $\mu_2 < \mu_1$, then, at the value $y_3$ calculated by the formula (45), the expected growth of TC budget revenues will be the minimum possible for a given standard deviation $\sigma_d$. If condition (43) is violated, then the solution of equation (42) requires the following inequality:

$$[D_1 D_2 (D_1 D_2 + D_1 D_3 + D_2 D_3) (\mu_2 - \mu_1)^2 + D_1 D_2 (D_1 + D_2)^2 (\mu_3 - \mu_1)^2)^2 \geq [(D_1 D_2 + D_1 D_3 + D_2 D_3)^2 (\mu_2 - \mu_1)^2 - (D_1 + D_2)^2 \times (D_1 D_2 + D_1 D_3 + D_2 D_3) (\mu_3 - \mu_1)^2] \times [(M_3 - M_1)^2 (D_1 + D_2)^3 - (D_1 + D_2)^3 D_1 D_2) - (\mu_2 - \mu_1)^2 D_1^2 D_2^2].$$  \hspace{1cm} (49)

From inequality (49) we can derive the condition by value $K = \frac{(M_3 - M_1)}{(M_2 - M_1)}$:

$$D_1^2 D_2^2 [D_1 D_2 + D_1 D_3 + D_2 D_3 + K^2 (D_1 + D_2)^2)^2 \geq [(D_1 D_2 + D_1 D_3 + D_2 D_3)^2 - (D_1 + D_2)^2 (D_1 D_2 + D_1 D_3 + D_2 D_3) + + (D_2 D_3) K^2] \times [K^2 (D_1 + D_2)^2 (D_1 D_1 + D_1 D_2 - D_1 D_2) - D_1^2 D_2^2].$$  \hspace{1cm} (50)

Condition (50) can be considered as a condition for variation of TC budget revenues without taking into account intergovernmental transfers $D_d$. In addition, expression (50) is a limitation of the level of allowable risk of TC budget revenues without taking into account intergovernmental transfers.

According to table 1, the proposed model is a tool for forecasting the results of financial decisions of local governments on the formation of the optimal structure of community budget revenues without taking into account intergovernmental transfers and forecasting the maximum possible level of their growth.
### Table 1. Scenarios for the development of territorial community finances to maximize the level of growth of community budget revenues at a given level of budget risk.

<table>
<thead>
<tr>
<th>Components of TC budget revenues excluding intergovernmental transfers</th>
<th>Optimal shares of TC budget revenue components</th>
<th>Variation of TC budget revenue components</th>
<th>Given level of risk on revenues of the TC budget</th>
<th>The maximum level of growth of TC budget revenues, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_1$</td>
<td>0.4</td>
<td>0.02</td>
<td>0.02</td>
<td>40</td>
</tr>
<tr>
<td>$y_2$</td>
<td>0.3</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_3$</td>
<td>0.3</td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario 2</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$y_1$</td>
<td>0.5</td>
<td>0.02</td>
<td>0.03</td>
<td>70</td>
</tr>
<tr>
<td>$y_2$</td>
<td>0.1</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_3$</td>
<td>0.4</td>
<td>0.04</td>
<td></td>
<td></td>
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<tr>
<td>Scenario 3</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$y_1$</td>
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<td>0.02</td>
<td>0.04</td>
<td>30</td>
</tr>
<tr>
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<td>$y_3$</td>
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<td>0.04</td>
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<td></td>
</tr>
<tr>
<td>$y_1$</td>
<td>0.4</td>
<td>0.02</td>
<td>0.05</td>
<td>60</td>
</tr>
<tr>
<td>$y_2$</td>
<td>0.5</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_3$</td>
<td>0.1</td>
<td>0.04</td>
<td></td>
<td></td>
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<tr>
<td>Scenario 5</td>
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<tr>
<td>$y_1$</td>
<td>0.5</td>
<td>0.02</td>
<td>0.06</td>
<td>20</td>
</tr>
<tr>
<td>$y_2$</td>
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<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_3$</td>
<td>0.2</td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Results.** The developed approach makes it possible to take into account variations in the components of TC budget revenues without taking into account intergovernmental transfers and expected levels of their growth while forming the optimal structure of community budget revenues. This approach can also determine the maximum possible growth of TC budget revenues for a given target level of budget risk.

The developed model is based on the resulting indicators of domestic financial decentralization and analytical data obtained during the analysis of structural imbalances in the revenues of budgets of territorial communities and the identification of budgetary risks. We set the economic and mathematical tasks for the first time. It allows us to solve the tasks step by step and form the directions for the strategic development of finances of territorial communities in the future. We staged the modeling in order to more effectively control the process of making strategic decisions to ensure the development of the finances of local communities.

Accordingly, we propose the following steps:

- outlining the problem to be solved - the formation of the optimal structure of budget revenues of territorial communities without taking into account intergovernmental transfers;
- generating alternative actions to solve the problem - modeling the optimal structure of budget revenues of territorial communities without taking into account intergovernmental transfers;
- development of methods for forecasting the consequences of each alternative financial solution - modeling scenarios for the formation of the optimal structure of budget revenues of territorial communities without taking into account intergovernmental transfers;
- development of a methodology for determining the probabilities for assessing the consequences of the decision - calculation of the degree of budgetary risk for each simulated scenario;
- determination of the criteria of financial decision, which reflects the choice of the most optimal alternative, the value of the components of revenues (tax revenues, non-tax revenues, income from capital transactions) of the budget of territorial communities without intergovernmental transfers at a certain level of budgetary risk.

For the first time, we have developed a scientific and methodological approach to forming the optimal structure of budget revenues of territorial communities without intergovernmental transfers and forecasting the maximum possible level of their growth under the control of budgetary risks. This allows local governments to generate multiple scenarios for vectors
of development of their own finances based on the optimal revenue structure of its budget and evaluate the results of each scenario, in particular, based on quantitative estimates of future revenues of local communities and budget risk.

DISCUSSION

In the scientific literature, there was no study of ways to optimize the structure of budget revenues of local communities in the context of taking into account and controlling the level of budgetary risks using the tools of economic and mathematical modeling.

At the same time, Yumit Karagozlu [15], developed a model for assessing the riskiness of local budgets, based on the calculation of coefficients of variation: «... determining the risk of planning budget indicators (planning risk factor); assessment of the risk of execution of budget indicators (coefficient of risk of execution); taking into account the influence of risk-forming factors (coefficient of influence of risk-forming factors)» [15]. According to the scientific beliefs of Yumit Karagozlu [15], statistical assessment of local budget risk by planning and execution risk coefficients involves the calculation of variance, standard deviation, and coefficient of variation for performance indicators of revenue and expenditure parts of the budget. The use of statistical methods in economic and mathematical modeling is effective in the analysis and evaluation of different scenarios and taking into account different risk factors within one approach, but the use of probabilistic characteristics narrows the practice of their implementation. At the same time, methods of simulation and probability assessment give a simplified statistical assessment of the probability of budget risks or non-fulfillment of budgets of local communities and allow analyzing of various probable events of budget risk, and this is a disadvantage, as such events can complicate practical mathematical calculations. In addition, computer simulation in experiments with mathematical models of complex real-world systems, although it allows us to obtain empirical estimates of the degree of influence of various factors on the level of budget risk but requires significant resources and is not always possible in practice. In turn, the technology "Risk Metrics", which was developed by J. P. Morgan "involves determining the degree of risk impact on the event by calculating the "risk measure " , namely the maximum possible potential change in the price of the portfolio, consisting of a different set of financial instruments, with a given probability and for a given period of time. Three main methods are used to calculate the "risk measure": analytical (or variation-covariance method), historical modeling, and statistical modeling (Monte Carlo method), which allow, based on the use of effective mathematical modeling tools, to obtain objective information about the level budgetary risk. However, this method is characterized by the complexity of calculations and the selection of parameters to assess this risk. Accordingly, in order to assess the structure of budget revenues of territorial communities and form proposals on this basis for their optimization in the context of permanent control of budgetary risks, we have developed a scientific and methodological approach using economic and mathematical modeling tools, based on J. Hicks' portfolio approach, which is not difficult to be calculated and is the most adapted to our defined mathematical problem.

The theoretical and methodological approach developed by us to quantify the structure of budget revenues of territorial communities without taking into account intergovernmental transfers in terms of tax revenues, non-tax revenues, and revenues from capital transactions takes into account the level of budget risk. This allows structuring the process of identifying and assessing these risks and is based on empirical assessments of revenue generation processes. The practical application of the proposed approach allows to model scenarios of budget risk, calculate their probabilities and obtain quantitative estimates of possible losses of revenues of local budgets in terms of tax revenues, non-tax revenues, and revenues from capital transactions in simulated scenarios.

CONCLUSIONS

Permanent crises affect the volume and structure of budget revenues of territorial communities, causing their significant variation. At the same time, the inability to predict the probability of occurrence and the level of budget risk have a negative impact on the course and effectiveness of financial decentralization. Therefore, it is necessary to develop an economic and mathematical apparatus to form the optimal structure of budget revenues of territorial communities in terms of tax revenues, non-tax revenues, and capital revenues and create prerogatives to identify the level of budgetary risks, based on their empirical assessment and measurement.

The developed scientific and methodological approach to the formation of the optimal structure of budget revenues of territorial communities in assessing the probability of budget risk allows to empirically measure it and predict scenarios of its occurrence, preventing negative trends in the likelihood of a significant decline in budget revenues.
Thus, we can conclude that in the process of modeling we have received the latest tools for financial decision-making using the portfolio approach of J. Hicks to form the optimal revenue structure of the territorial community budget, taking into account different types of budgetary risk at a certain predicted by mathematical calculations level. The proposed tools allow local governments to generate alternative financial solutions and evaluate the results and consequences of their possible adoption in determining the strategic guidelines for the development of their finances.

In cases where in practice a large number of scenarios will be modeled to form the optimal structure of budget revenues of territorial communities in terms of tax revenues, non-tax revenues, revenues from capital transactions at a controlled level of budget risk, local governments may face uncertainty in choosing a specific scenario. Therefore, in order to decide on the choice of the most optimal of them, in our opinion, is to use the method of financial decision-making based on the principle of the central situation. In this case, we choose a dominant situation from a set of existing scenario alternatives, in which the total significance of the existing criteria for the effectiveness of the financial decision is maximum, and the total significance of the criteria of other scenarios other than the central is minimal. In essence, the principle of the central situation is that the financial decision is made on the basis of the priority of central financial decision is maximum, and the basis for further research, development, and research in this area.

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Кнейслер О., Спасів Н., Маринчак Л.

НОВІТНІ ІНСТРУМЕНТАРІЙ ФОРМУВАННЯ ОПТИМАЛЬНОЇ СТРУКТУРИ ДОХОДІВ БЮДЖЕТІВ ТЕРИТОРІАЛЬНИХ ГРОМАД

Досліджено вплив бюджетних ризиків на формування доходів бюджетів територіальних громад і розроблено мето- дичний підхід до оцінювання структури доходів бюджетів територіальних громад в умовах фінансової децентралізації. Зазначено, що сьогодені особливого значення набуває впровадження в практику новітніх методик щодо оптимізації доходів бюджетів територіальних громад за перманентного контролю за рівнем бюджетних ризиків. Саме математичне прогнозування ймовірності настання бюджетних ризиків і його емпіричне вимірювання потребує особливої уваги. Розглянуто основні підходи вітчизняних і зарубіжних науковців до прогнозування ймовірності настання бюджетних ризиків за допомогою економико-математичного інструментарію, використання якого на практиці, з одного боку, забезпечить його перманентний контроль, а з іншого — дасть змогу оптимізувати доходи бюджетів територіальних громад. Установлено, що існуючі методики складно імплементувати у вітчизняну практику в силу їхнього значного математичного обрання. Проведено розрахунки за використання постулатів портфельної теорії, з одного боку, забезпечить їх підтримання формування оптимальної структури доходів бюджету територіальних громад у розрізі загального та спеціального фондів та ідентифікувати напрями мінімізації бюджетних ризиків із метою забезпечення оптимальної структури доходів без урахування міжбюджетних трансфертів в умовах перманентних кризових явищ, що дозволить органам місцевого самоврядування територіальних громад підвищити виконання показників доходів їхніх бюджетів. Шляхом застосування інструментарію економико-математичного моделювання отримано новітній інструментарій прийняття фінансових рішень із використанням портфельного підходу Дж. Хікса щодо формування оптимальної структури доходів бюджету територіальних громад без міжбюджетних трансфертів із урахуванням рівня бюджетного ризику. Зроблено висновок, що розроблені новітній інструментарій до формування оптимальної структури доходів бюджету територіальних громад за перманентного прогнозування максимально можливого рівня їхнього зростання при умові контролю бюджетних ризиків в разі їхньої ідентифікації дає змогу оцінювати шлюхи мінімізації, формуючи множину сценаріїв щодо оптимальної структури майбутніх доходів бюджету бюджету та бюджетного ризику. Установлено, що запропонований інструментарій дає змогу генерувати альтернативні фінансові рішення органам самоврядування територіальних громад та оцінювати результати й наслідки можливого їхнього прийняття за визначення стратегічних орієнтирів розвитку фінансів територіальних громад. За- пропоновано в подальшому використовувати розроблений науково-методичний підхід щодо оптимізації структури доходів бюджетів територіальних громад у контексті настання різного роду бюджетних ризиків, що, безумовно, дозволить органам місцевого самоврядування ухвалювати ефективні фінансові рішення при розробці програм соціально-економічного розвитку територіальних громад.

Ключові слова: фінансова спроможність територіальної громади, фінанси територіальних громад, бюджет територіальної громади, фінансова децентралізація, фінансова система, бюджетний ризик, місцеві запозичення, доходи бюджету територіальної громади

JEL Класифікація: N72, N77