ECONOMIC AND MATHEMATICAL MODELING OF THE FUNCTIONING OF AN INDUSTRIAL ENTERPRISE

ABSTRACT

The purpose of the article is to model the activities of an industrial enterprise as an independent economic unit in order to increase the profit of the enterprise in terms of economic efficiency.

Method. In the course of the research the methodology of economic-mathematical modeling was applied. The enterprise is considered as a complex system – a sublimated enterprise, an economic object. The use of simulation methods on specific statistical material of the industrial enterprise «PivdGOK» is given. The application of the methods of digital economy allowed to obtain the relevant indicators of the enterprise and to obtain appropriate visual and graphical representations of the results.

Results. The economic and mathematical model of the industrial enterprise of PJSC «PivdGOK» is constructed, it allowed to generalize results of activity of the industrial enterprise. Analytical relations concerning the activity of an industrial enterprise are obtained, namely a systematic study of the relationship between costs, volume of activity and profit of the enterprise.

Scientific novelty. For the first time the economic-mathematical model of activity of the industrial enterprise in the sense of definition of its effective functioning as economic unit is constructed. Obtained in the implicit form of the dependence of profit at a given economic efficiency allowed us to investigate this relationship in order to build a simulation model.

Practical significance. The use of Mathcad software allowed to building a simulation model of the industrial enterprise. An analytical expression of the connection between economic efficiency and profit is obtained, which allows to carry out a corresponding analysis of the activity of an industrial enterprise. It is shown under what conditions it is possible to achieve the greatest profit at a given economic efficiency. The use of digital modeling allowed to numerically obtain the optimal values of profit and graphically display the results. Graphical representation of the obtained results allows for visual analysis.

Keywords: model, profit, enterprise, activity, resources, economic object

JEL Classification: C19, D29

INTRODUCTION

In market conditions, the center of economic activity has shifted to the main section of the entire market economy – the enterprise [1]. It is at this level that the necessary goods are produced for society and necessary services are determined. The enterprise employs the most qualified personnel. There, issues of efficient resource consumption, application of high-performance machinery and technology are settled. The enterprise organizes the production process, develops strategic, current and operating plans, and conducts effective management. The enterprise performs innovation and investment activities and takes measures to economically use financial resources. In the market economy, the only enterprise that can survive is the one that determines requirements of the market in the most competent way, builds and organizes production of goods that are in demand, provides high incomes for the most qualified employees.
The purpose of the enterprise is to meet public needs and make a profit [2]. Naturally, one should agree that business is the economic activity of business people, their art and ability to bring in increasing profits to the enterprise, ensure a high level of efficiency. In an open economy, business development contributes to saturation of the consumer market with goods and services, activates economy restructuring, stimulates introduction of scientific and technological achievements, acts to raise production efficiency to its maximum values. Technologies used at the enterprise play a significant role and impact greatly all components of the internal environment of the enterprise and are interrelated with them. An economic component is a totality of economic processes that include capital and cash flows, economic indices of the enterprise. Among factors of the internal environment of enterprises, a special place belongs to the information component – a set of organizational and technical means that provide channels and networks of the enterprise with relevant information for effective communication in enterprise management. With the advent and development of information networks, including the Internet, the success of the enterprise is increasingly determined by the level of information technologies applied. The result of business people's production activities is selling products (works, services) to the consumer and making a certain amount of money. The financial result of the enterprises the difference between earnings and production costs. Effective management of the enterprise requires knowledge of its features which allow predicting its behavior during changes of both the external and internal environments. Economic and mathematical modeling of the enterprise operation is one of the possible ways to solve the problem [3]. Such a model allows focusing on those sides of the enterprise that are significant in current conditions.

LITERATURE REVIEW

Analysis of results of researches and publications on activities of enterprises shows that in most cases they do not pay sufficient attention to economic and mathematical modeling of enterprise operation which is common for enterprises, regardless of their specific activities [4; 5]. Thus, in [6] the main attention is paid to economic activities of industrial enterprises. Investigations in [7–9] deal with a comprehensive assessment of efficiency of enterprise development. In [10], which is of particular interest, the authors use well-known laws of physics and elements of fuzzy mathematical logic to determine some «economic laws, e.g. the laws of duration of economic cycles and power of economic crises» (courtesy translation), thus applying a new approach called fuzzy physical economics. It is important that this work advances the idea of «whitening the black box» which is widely used in cybernetics [11]. Particular attention is attracted by [12], in which the authors carefully study the model «costs – volume – profit» (CVP-approach) of economic management of the enterprise as a method of system research of the relationship of costs, volume of activity and profits of the enterprise. This model highlights that the cost management process is closed and based on the main principle of cost control – cost management aimed at reducing costs. [13; 14] investigate issues related to government regulation of economy. In particular, they study economic efficiency of government regulation and determine its shortcomings caused by limitations, incompleteness of information, bureaucracy, inability of government structures to fully anticipate consequences of political decisions made, primarily long-term ones.

AIM

Therefore, in current conditions, sufficient attention should be paid to development of new, more efficient and adequate to modern economic realities approaches to managing the financial result of the enterprise. One of possible ways to solve these problems involves economic and mathematical modeling of the enterprise as a complex economic entity in order to increase its profits in conditions of specified economic efficiency [15–19].

RESULTS

Economy comprises a variety of enterprises that differ both qualitatively and quantitatively. Then a sublimated enterprise can to some extent be called a primary economic entity (EE). However, it should be emphasized that the EE, despite being the «backbone» of economy, has a rather complex structure due to the nature of tasks it performs. Considering the fact that the EE is an open system, financial means (generally determined as cost inputs and laying the foundation of the EE activities) come to its input. Economic analysis shows that cost efficiency is a function of real cost inputs and can be approximated by the function

\[ E_1 = \frac{C_1}{C} = e^{-\alpha C}, \]

(1)
Where \( C \) is the cost input, dollar equivalents; \( C_i \) is the real cost input, dollar equivalents; \( \alpha \) is the parameter characterizing a drop-in cost efficiency, 1/dollar equivalent.

Efficiency of standard working capital (SWC), which is determined as the SWC value-real cost input relation, is a function of the real cost input and can be written as follows

\[
E_2 = \frac{C_2}{C_1} = 1 - e^{-\beta C_1}
\]  (2)

where \( C_2 \) is the SWC value, dollar equivalents; \( \beta \) is the parameter characterizing SWC efficiency increase, 1/dollar equivalent.

Income efficiency is determined as the relation of the income value to the SWC value

\[
E_3 = \frac{D}{C_2} = q,
\]  (3)

where \( D \) is the income value, dollar equivalents; \( q \) is income efficiency.

According to (1), (2) and (3) the EE income values determined by the formula

\[
D = C \cdot e^{-\beta C} \cdot (1 - e^{-\beta C_i e^{-\alpha C_i}}) \cdot q.
\]  (4)

Thus, (4) determines the economic and mathematical model of the EE. In terms of cybernetics, this model is a «grey box» for which the structure is determined but the parameters included in this structure are unknown.

Analysis of the EE model, the structure of which is set by (4), shows that it depends on three parameters \( \alpha, \beta \) and \( q \). To find the parameter values, i.e. to identify model (4), statistics that connects the input variable \( C \) and the output variable \( D \) should be used. Thus, ones hold have the pairs

\[
(C_i, D_i), \quad (i = 1, \ldots, n),
\]  (5)

which are connected by equation (4).

By substituting (5) into (4), the system of non-linear equations with unknown parameters is received

\[
C_i \cdot e^{-\alpha C_i} \cdot (1 - e^{-\beta C_i e^{-\alpha C_i}}) \cdot q = D_i, \quad (i = 1, \ldots, n).
\]  (6)

Since in general the number of equations is greater than the number of unknowns \( (n > 3) \), system of equations (6) is incompatible. Therefore, for its solution, the least squares method (LSM) is used, which consists in minimizing the sum of squares of deviations of the left parts from the right parts of equations (6) relative to the parameters \( \alpha, \beta \) and \( q \)

\[
F(\alpha, \beta, q) = \sum_{i=1}^{n} \left( C_i \cdot e^{-\alpha C_i} \cdot (1 - e^{-\beta C_i e^{-\alpha C_i}}) \cdot q - D_i \right)^2 \rightarrow \min_{\alpha, \beta, q}
\]  (7)

Optimal values of the parameters \( \alpha^*, \beta^*, q^* \) are determined by minimizing functional (7). As a result, EE model (4) takes the form

\[
D = C \cdot e^{-\beta^* C} \cdot (1 - e^{-\beta^* C_i e^{-\alpha^* C_i}}) \cdot q^*.
\]  (8)

Economic efficiency of the EE model is determined by the formula

\[
E = \frac{D}{C} = e^{-\beta^* C} \cdot (1 - e^{-\beta^* C_i e^{-\alpha^* C_i}}) \cdot q^* - 1.
\]  (9)

The resulting model (8) allows the formula for calculating the profit of the EE

\[
P(C) = C(e^{-\alpha^* C} \cdot (1 - e^{-\beta^* C_i e^{-\alpha^* C_i}}) \cdot q^* - 1).
\]  (10)
Using (9), one can formulate a problem about optimizing management of the EE in order to achieve the greatest profit at specified economic efficiency. Considering (9) and (10), this problem is mathematically written as follows

\[
\begin{aligned}
\min_C & (e^{-\alpha^*C} \cdot (1 - e^{-\beta^*C}e^{-\alpha^*C}) \cdot q^* - 1) \\
\text{s.t.} & e^{-\alpha^*C} \cdot (1 - e^{-\beta^*C}e^{-\alpha^*C}) \cdot q^* - 1 \geq E
\end{aligned}
\]  

(11)

where \(E\) is restrictions on the lower boundary of economic efficiency of the EE.

A rather complex type of function (10) causes the need to use numerical methods to find the maximum, which greatly complicates solution of the problem.

As an example, let us consider the problem of maximizing profits at the PJSC «PivdGZK». Figure 1 plots statistical data on economic indices at the PJSC «PivdGZK» [20; 21].

To build an economic and mathematical model of the PJSC «PivdGZK» according to Figure 1, (4) is used as a structure. For identification purposes, functional (7) is minimized. Considering that the functional is non-linear relative to the parameters, the Minimize program, which is part of the Mathcad software package, is applied. The calculations result in obtaining optimal values of the parameters

\[
\alpha^* = 0.00807, \quad \beta^* = 0.68551, \quad q^* = 1.442.
\]

(12)

Considering (12), the obtained model (8) is presented in the following form

\[
D = 1.442 \cdot C \cdot e^{-0.00807 \cdot C}(1 - e^{-0.68551 \cdot C}e^{-0.00807 \cdot C}).
\]

(13)

At the same time, the index of statistical data determination in relation to the resulting model is calculated by the formula

\[
R^2 = 1 - \frac{\sum_{i=1}^{50}(D_i - \hat{D})^2}{\sum_{i=1}^{50}(D_i - \bar{D})^2},
\]

(14)

where \(\hat{D} = \frac{1}{50} \sum_{i=1}^{50} D_i\)

Substitution of numerical values in (14) gives the value of the determination index

\[
R^2 = 1 - \frac{0.212}{15.645} = 0.986.
\]

(15)
Given that the condition

$$R^2 = 0.986 > 0.9$$

is met, it can be argued that, according to the Chad dock scale, there is a very strong relation among variables [15].

Figure 2 presents statistical data and the results of calculations by formula (13).

![Figure 2. Statistics and results of calculations](image)

Analysis of Figure 2 confirms a rather good approximation of statistical data to the results of calculations according to (13), which is proved by determination index value (15).

According to (10), the profit is written in the following form

$$P(C) = C \cdot (1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C} \cdot e^{-0.00807 \cdot C}) - 1).$$

(16)

Figure 3 plots dependence of profit (16) on the cost input.

![Figure 3. Profit-cost dependence](image)

Analysis of Figure 3 demonstrates that the dependence under study is of an extremal nature. As is seen in Figure 3, during the period of change in costs up to 5 M UAH, there are losses that reach the largest amount of 0.32 M UAH when the costs amount to 0.742 M UAH. The calculations are performed using the Minimize program, which is part of the Mathcad software.
package, through solving the problem

\[ P(C) = C \cdot (1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C \cdot e^{-0.00807 \cdot C}}) - 1) \to \min. \]

According to (16), the largest profit can be found by solving the problem

\[ P(C) = C \cdot (1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C \cdot e^{-0.00807 \cdot C}}) - 1) \to \max. \]  

(17)

To solve (17), the Maximize function is used, which is part of the Mathcad software package. As a result,

\[ P_{\text{max}} = P(21.617) = 4.566 \text{MUAH}. \]  

(18)

According to (9), economic efficiency of the PJSC «PivdGZK» is written in the following form

\[ E(C) = 1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C \cdot e^{-0.00807 \cdot C}}) - 1. \]  

(19)

Figure 4 plots dependence of economic efficiency (19) on costs.

Analysis of Figure 4 demonstrates that the dependence under study is of an extremal nature. Using the Maximize function, which is part of the Mathcad software package, we find

\[ E_{\text{max}}. \]  

(20)

Along with that, the economic efficiency-profit relation is of interest as well. To investigate this relation, it is suggested to consider a system of two equations which is composed of profit (16) and economic efficiency (19)

\[ \begin{align*}
P(C) &= C \cdot (1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C \cdot e^{-0.00807 \cdot C}}) - 1) \\
E(C) &= 1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C \cdot e^{-0.00807 \cdot C}}) - 1.
\end{align*} \]  

(21)

In these equations, the cost input \( C \) acts as a parameter. If this parameter is excluded from system of equations (21), dependence of profit \( P \) on economic efficiency \( E \) is obtained. For this, system of equations (21) is written in the following form

\[ \begin{align*}
P &= C \cdot E \\
E &= 1.442 \cdot e^{-0.00807 \cdot C} \cdot (1 - e^{-0.68551 \cdot C \cdot e^{-0.00807 \cdot C}}) - 1.
\end{align*} \]  

(22)
Then, considering the first equation of system (21), the second equation is written as follows

$$E = 1.442 \cdot e^{-0.00807} \cdot \frac{P}{E} \cdot (1 - e^{-0.68551} \cdot e^{-0.00807} \cdot \frac{P}{E}) - 1. \quad (23)$$

Equation (23) implicitly determines dependence of profit $P$ on economic efficiency $E$. Thus, by specifying the value of economic efficiency $E$ through solving non-linear equation (23), the value of profit $P$ can be found. It should be noted that (23) is solved using the root function, which is part of the Mathcad software package.

Figure 5 plots the resulting dependence of profit on economic efficiency.

![Figure 5. Profit-economic efficiency dependence](image)

Analysis of Figure 5 demonstrates that the profit-economic efficiency dependence is «loop» shaped. In this case, two values of profit correspond to one value of efficiency.

In practice, the profit value at the specified value of economic efficiency is obviously more important. Figure 5 demonstrates that economic efficiency is restricted by the maximum value determined by (20), and the maximum profit value is restricted by the value of (18).

The problem of achieving the greatest profit considering restrictions on economic efficiency (11) and values of parameters (12) is formulated as follows

$$\begin{cases} C \cdot (1.442 \cdot e^{-0.00807} \cdot (1 - e^{-0.68551} \cdot e^{-0.00807} \cdot C)) - 1) \rightarrow \max \frac{C}{E} \\ 1.442 \cdot e^{-0.00807} \cdot (1 - e^{-0.68551} \cdot e^{-0.00807} \cdot C) - 1 \geq E \end{cases} \quad (24)$$

According to Figure 5, specificity of problem (24) is caused by the impact of the value of economic efficiency restrictions on its solution.

If the condition

$$E \leq E(P_{\max} = 4.566) = 0.211,$$

is met, the maximum profit is determined by the value

$$P_{\max} = 4.566 \text{ M UAH}$$

at the cost input of

$$C = 21.617 \text{ M UAH}.$$
When
\[ E(P_{\text{max}}) \leq E \leq E_{\text{max}}, \]  
(25)
the maximum profit value is determined as the result of intersection of the line
\[ E = E \]  
(26)
and the upper boundary of the «loop» (Figure 5). Considering (26), problem (24) is mathematically written as follows
\[ \begin{align*}
C & \rightarrow \max \\
1.442 \cdot e^{-0.00807C} \cdot (1 - e^{-0.68551C \cdot e^{-0.00807C}}) - 1 &= E
\end{align*} \]  
(27)
Thus, to solve (24) under condition (25), it is necessary to solve system (27).

The condition of the maximum costs is associated with the choice of the intersection point of the graph of function (26) with the upper boundary of the «loop» in Figure 5.

In our case, condition (25) is written in the following form
\[ 0.211 \leq E \leq 0.349. \]  
(28)
Let \( E = 0.3, \)
then (26) looks like
\[ \begin{align*}
C & \rightarrow \max \\
1.442 \cdot e^{-0.00807C} \cdot (1 - e^{-0.68551C \cdot e^{-0.00807C}}) - 1 &= 0.3
\end{align*} \]  
(29)
To solve equation (29), the root function, which is part of the Mathcad software package, is used. As a result, two roots are obtained
\[ C_1 = 4.037\quad C_2 = 12.805. \]  
(30)
According to (29), we choose a larger amount of costs, which corresponds to the intersection of the graph of function (26) with the upper boundary of the «loop» in Figure 5. According to (22), the largest profit makes
\[ P(12.805) = 12.805 \cdot 0.3 = 3.842\text{ MUAH} \]  
(31)
at the cost input of
\[ C = 12.805\text{ MUAH}. \]
Thus, depending on the restriction on the lower value of economic efficiency, one has the corresponding largest profit value.

**CONCLUSIONS**

Achieving the largest profit of an economic unites one of the most important problems conditioned by market relations. One of possible ways to solve this problem is application of the method of economic and mathematical modeling of the economic unit. Relying on the economic and mathematical model of the economic unit, the research demonstrates under what conditions the greatest profit can be achieved at specified economic efficiency. The suggested method is used for simulation on the specific statistical material of the PJSC «PivdGZK». Further research involves implementation of the model at enterprises.
REFERENCES / ЛІТЕРАТУРА

Методика. У процесі дослідження застосовано методологію економіко-математичного моделювання. Підприємство розглянуто як складну систему – сублімоване підприємство, економічний об’єкт. Наведено використання методів імітаційного моделювання на конкретному статистичному матеріалі промислового підприємства ПрАТ «ПівдГЗК». Застосування методів цифрової економіки дозволило отримати відповідні показники діяльності підприємства та отримати слушні візуально-графічні відображення отриманих результатів.

Результати. Побудовано економіко-математичну модель промислового підприємства ПрАТ «ПівдГЗК». Це дозволило узагальнити результати діяльності промислового підприємства. Отримано аналітичні співвідношення щодо діяльності промислового підприємства, а саме системного дослідження взаємозв’язку витрат, обсягу діяльності та прибутку підприємства.

Наукова новизна. Уперше побудовано економіко-математичну модель діяльності промислового підприємства щодо визначення його ефективного функціонування, як економічної одиниці. Отримані в неявній формі залежності прибутку за заданої економічної ефективності дозволило дослідити цей зв’язок задля побудови імітаційної моделі.

Практична значимість. Застосування програмного комплексу Mathcad дозволило побудувати імітаційну модель функціонування промислового підприємства. Отримано аналітичний вираз зв’язку економічної ефективності і прибутку, що дозволяє провести відповідний аналіз діяльності промислового підприємства. Показано, за яких умов можливо досягнення найбільшого прибутку за заданої економічної ефективності. Застосування цифрового моделювання дозволило чисельно отримати оптимальні значення прибутку за заданої ефективності та графічно відобразити отримані результати. Графічна представленисть отриманих результатів дозволяє провести візуальний аналіз.

Ключові слова: модель, прибуток, підприємство, діяльність, ресурси, економічний об’єкт

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