INTRODUCTION

The pension system is an important component of the population’s social protection in each country. The level of stability in society and the well-being of the population depend on its effectiveness. To the moment, the domestic pension system is not perfect. It has complex problems. To a certain extent they are caused by the drawbacks of the existing pension system and the imperfect mechanism of its financing. The problems of the public pension system are important for Ukraine in the context of the demographic crisis and the aging of the population. Moreover, reforming the Ukrainian pension system, growth of the Pension Fund of Ukraine income and balancing its budget are rather deep problems for the state pension insurance. There is a budget deficit because in recent years the volume of revenues is insufficient to meet the projected expenditures. Therefore, the topic of this research is, most likely, relevant, important, and accepted for further elaboration.

ABSTRACT

The paper covers the problems of the domestic system of state pension insurance. Using the example of the Pension Fund of Ukraine, the influence of certain macroeconomic, demographic, financial and economic factors on the formation of the revenue part of the budget of the respective fund is investigated by the method of mathematical modeling. It is identified that the following endogenous indicators, whose changes should be under special attention of the state when forecasting the income of the Pension Fund of Ukraine, will serve as determining parameters: average monthly wage dynamics and inflation (macroeconomic indicators), the amount of the single premium, which is divided into compulsory state pension insurance (financial and economic index), and the number of the employed population (demographic index). The existence of multicollinearity between the selected factor variables was tested using the Farrar – Gloeber test. It is proved that since there is no high degree of correlation between factor variables, it is expedient to carry out the parameterization step using the method of least squares (MLS). Based on the constructed econometric model of multiple regression, while maintaining the average salary, the growth rate of the revenue side of the budget of the Pension Fund of Ukraine will be 0.41, taking into account the synergistic effect of other factors considered (growth rate of single contribution, growth rate of employed population, and inflation rate). While maintaining the rates of insurance premiums for employers in the funds of compulsory state social insurance of Ukraine, the growth rate of the revenue side of the budget of the Pension Fund of Ukraine will be 0.73 taking into account the synergistic effect of other factors under consideration (growth rate of average monthly wage, growth rate of employed population, and inflation rate). While maintaining the number of the employed population, the growth rate of the revenue part of the budget of the Pension Fund of Ukraine is 0.86 taking into account the synergistic effect of other factors under consideration (average monthly wage growth rate, single contribution rate, and inflation rate).

Keywords: state pension insurance, mathematical methods, econometric models, correlation-regression analysis, information and communication technologies in the sphere of economy

JEL Classification: C31, G23

MATHEMATICAL MODELING

PENSION FUND OF UKRAINE BY METHOD OF DIAGNOSIS PROFITABLE PART OF THE
LITERATURE REVIEW

Existing problems of the system of state pension insurance have become an important aspect of the research of scientists and are at the center of attention of many specialists: Blake [1], Crossley, Jametti [2], Müller, Schiess [3], Atalay, Barrett [4], Brinch, Hernæs and Jia [5], Thomas, Spataro [6], Champagne, Chrétien, Coggins [7], Goergen, O’Sullivan, Wood, Baric [8], Milligan and Schirle [9]. Considered the problems of domestic development of the compulsory pension system in its phased context (Shaliyevska [10], Sydorchuk [11], Gudz, Yankovskyj [12], Maleczka [13]). L. Vasechko [14] has examined the dependence of the dynamics of the revenue part of the Pension Fund of Ukraine from the effect of internal economic factors on the region context by the method of correlation-regression analysis based on longitudinal data, N. Lavruk and N. Hrendey [15] analyzed the dynamics of the Pension Fund of Ukraine revenues and its income structure and based on modeling and forecasting methods they projected the trend of the revenue part of its budget for future periods. V. Bredyuk and O. Dzhoshi [16] elaborated an economic and mathematical model of the dynamics of the pension system in Ukraine taking into account the increase in the retirement age for women, and based on that a forecast of the share of pension system in GDP has been made.

OBJECTIVE

To quantify and analyze the impact of endogenous financial and economic factors on the formation of the income part of the Pension Fund of Ukraine is the purpose of this article.

RESEARCH METHODOLOGY

It is impossible to imagine modern science without the extensive use of mathematical modeling. Not surprisingly, the methodology of mathematical modeling is rapidly evolving, covering all new areas – from the development of technical systems and their management to the analysis of the most complex economic and social processes. Nowadays, mathematical modeling is an important stage of development – it is embedded in the structures of the so-called information society, and without the possession of information resources, one cannot even imagine the solution of the large-scale problems facing the world community. The problem of the application of mathematical methods in economics has been investigated by the following scholars: Gorkunenko, Lupenko, Luczkiv [17] and others. A number of works by both domestic and foreign authors [18] are devoted to the practical application of information technologies for the implementation of the mathematical apparatus in the fields of finance and economy. While studying various processes and phenomena in the economy, models of multiple regression allowing to take into account the influence of all factors on the productive variable are most often used. We will use this model in our study.

RESULTS AND DISCUSSION

The purpose of the study is to establish and investigate the dependence of the own revenues of the insurance Pension Fund of Ukraine on the influence of the following internal (endogenous) factors.

- **Macroeconomic:** average monthly wage dynamics and inflation;
- **Financial and economic:** the amount of the single premium, which is divided into compulsory state pension insurance;
- **Demographic:** the number of the employed population.

And also, on the basis of established correspondences, to determine the main macroeconomic, demographic, financial and economic parameters, whose changes should be under special attention of the state when forecasting the income of the Pension Fund of Ukraine and, consequently, strengthening the financial sources of obligatory state social insurance of Ukraine.

The selection of factors included in the mathematical model is made on the basis of the purpose of the study, the prerequisites for the possible correlation between them and the availability of statistical information to the extent sufficient to perform the mathematical modeling and to carry out appropriate correlation-regression analysis.

A review of the economic literature shows that the amount of the single social contribution allocated to compulsory state pension insurance has a significant impact on the income part of the budget of the Pension Fund of Ukraine, since contributions from the single social contribution are the main source of replenishment of the Pension Fund of Ukraine. The level of inflation also has a significant impact on PFU revenue, because inflationary processes threaten with the depreciation or
loss of retirement assets.

The number of employed population, since only the insured employed population makes payments to the Pension Fund of Ukraine, is another important factor. With the increase in the number of employed people, revenues to the Pension Fund of Ukraine should increase as well. Conversely, reducing the number of employed people will reduce the base of contributions to the budget of the Pension Fund of Ukraine. We believe that the number of employed people is a better indicator than the number of unemployed, because an essential part of the unemployed are not registered in employment centers, while businesses usually keep records for the vast majority of employees. The widespread cases of shadowing a part of an employee’s wages in order to reduce the deductions to the budgets of the Pension Fund of Ukraine are taken into account, but are not reflected statistically.

The statistical data used in the study is provided by the M. V. Ptukha Demography and Social Research Institute of NASU as well as received from the official site of the State Statistics Service of Ukraine: www.ukrstat.gov.ua.

The initial step to building an econometric model is to identify the variables. According to the results of identification, we get: \( Y \) – own income of the Insurance Pension Fund of Ukraine, mln. UAH (in constructing the model we will use a dimensionless amount that characterizes the rate of growth of the annual own income of insurance the Pension Fund of Ukraine); \( X_1 \) – growth rate of the average monthly wage; \( X_2 \) – growth rate of the single contribution; \( X_3 \) – growth rate of the number of employed population; \( X_4 \) – inflation rate.

It should be noted that the use of digital technologies greatly simplifies the process of constructing and researching mathematical models of economic processes (Figure 1). To solve this problem, we will use MS Office Excel general-purpose software [19].

![Figure 1. Dynamics of change of econometric model indicators](image)

As we can see, these indicators have a small coefficient of variability. Therefore, in a theory, they should have little impact on significant changes in the resultant variable. Nevertheless, they make a significant contribution to the revenue part of the Pension Fund of Ukraine.

Figure 2 shows the change of another indicator of the econometric model – the level of inflation in comparison with other factors of the model.

Obviously, this indicator is more dynamic and its maximum «bursts» are in 2008 (22%), 2014 (25%) and 2015 (43%).

The model specification is the second stage of construction, it involves establishing the form of correlation between the resultant and factor variables. We will construct a correlation field for the dependence of the employment rate of the population on each of the factors using a point diagram in MS Excel.

We use the trend line to determine the best type of relationship. Using the Trend Line Format dialog box, we determine the coefficient of determination for each of the diagrams. Comparing the coefficients of determination \( R^2 \) for each type of dependencies (linear, exponential, logarithmic, etc.), we choose optimal ones, i.e. those for which the value \( R^2 \) becomes the maximal of the all possible.
Based on the above study, it was established that there is a linear relationship between the relevant factors of the econometric model:

\[ Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + u, \]  

(1)

where \( u \) – a random or probabilistic component that is not calculated directly from the equation.

The next step in model building is the parameterization step: finding parameter estimations \( \hat{a}_i \ (i = 0...4) \) and constructing an appropriate regression equation. The Least Squares Method (LSM) enables this step to be implemented. In the absence of multicollinearity, heteroscedasticity, the estimates found with LSM are unbiased, reliable, and efficient.

Let us check for the presence of multicollinearity between the selected factor variables using the Farrar – Glober test. If the existence of multicollinearity between the factors is confirmed, we should draw conclusions about the variants of its elimination and the possibility of using LSM (least squares method) to estimate the model parameters [20].

Using the Farrar – Glober algorithm, we find out whether there is a strong correlation between the factor variables.

In the first step of this algorithm we normalize the factor variables \( X_1, X_2, X_3, X_4 \) of this econometric model by the formula:

\[ x_{ij} = \frac{(x_{ij} - \bar{x}_j)}{\sqrt{n(\bar{x}_j^2 - (\bar{x}_j)^2)}}, \quad i = 1, n; \quad j = 1, m, \]  

(2)

where \( n \) – the number of observations of the relevant factor variables; \( m \) – the number of factor variables.

We build a matrix \( X^* \), whose elements are values \( x_{ij} \) and the corresponding transposed matrix \((X^*)^T\). We calculate the correlation matrix (Table 1) by the formula

\[ R = (X^*)^T \cdot X^*; \]

**Table 1.** The correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>1,343.64</th>
<th>243.83</th>
<th>1,015.21</th>
<th>1.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>243.83</td>
<td>462.01</td>
<td>386.82</td>
<td>-1.07</td>
<td></td>
</tr>
<tr>
<td>1,015.21</td>
<td>386.82</td>
<td>9,852.32</td>
<td>-22.96</td>
<td></td>
</tr>
<tr>
<td>1.90</td>
<td>-1.07</td>
<td>-22.96</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>

Hence the determinant of the correlation matrix \( det \ R = 218,357,249.73 \) and value of \( \chi^2 \) – criterion will be equal:

\[ \chi^2 = - \left[ n - 1 - \frac{1}{6} \cdot (2m + 5) \right] \cdot \ln|R| = -208.02 \]

(3)

where \( n \) – number of observations \( (n = 11) \), \( m \) – number of factor variables \( (m = 4) \).
Comparing the $\chi^2$ value obtained with the table value $\chi^2_{\text{tabl}} = 12.59$ while $\frac{1}{2} m (m - 1)$ degrees of freedom and a given level of significance $\alpha (\alpha = 0.05)$, we conclude: because $\chi^2 > \chi^2_{\text{tabl}}$, then there is no multicollinearity phenomenon in this array of factor variables.

Since there is no high degree of correlation between the factor variables, we can carry out the parameterization step relying on LSM in MS Excel using the formulas

$$\hat{A} = (X^T X)^{-1} \cdot X^T Y,$$

(4)

where $\hat{A}$ – vector column of equation coefficient estimates; $X^T$ – transposed matrix to matrix $X$; $(X^T X)^{-1}$ – inverted matrix to the product of two matrices $X^T X$.

As a result of the calculations, an economic and mathematical model was obtained.

$$\hat{Y} = -1.00 + 0.41X_1 + 0.73X_2 + 0.86X_3 + 0.001X_4.$$  

(5)

The constructed model is adequate, as evidenced by the average value of the relative error of approximation $A_i$

$$\bar{A}_i = \frac{1}{n} \sum A_i, \quad A_i = \frac{|y_i - \hat{y}_i|}{y_i} \cdot 100\%,$$

(6)

which is within 10% and is equal to $\bar{A} = 5.91\%$. Another criterion for model adequacy is related to the value of the coefficient of determination $R^2$

$$R^2 = 1 - \frac{\sum\limits_{i=1}^{n} e_i^2}{\sum\limits_{i=1}^{n} y_i^2} = 1 - \frac{\bar{e}^2}{\bar{y}^2}.$$  

(7)

The closer $R^2$ to one, the more significant the relationship between the variables, i.e. the change in the resultant variable is largely explained by the change in the factor variable, and only a small fraction of the change is explained by other factors. If $R^2 \rightarrow 0$, then the factor variable is not influential, and therefore the model is not adequate to the phenomenon. For the built model $R^2 = 0.95$.

The coefficients of elasticity $E_i$ and overall elasticity $E$ are equally important for the analysis and interpretation of the results. The elasticity coefficients $E_i$ are calculated for each factor variable $X_i$: $E_i = \hat{a}_i \cdot \frac{\bar{x}_i}{\bar{y}}$, $(i = 1, 5)$ and show how many percent will change $Y$ (will increase at $E_i > 0$ and decrease at $E_i < 0$), if the corresponding value $X_i$ will increase at 1%. The overall elasticity $E (E = \sum_{i=1}^{5} E_i)$ shows how many percent the result will change $Y$, if at the same time increase by 1% all factors $X_i$.

The corresponding values of the coefficients of elasticity of the factor variables of the studied regions are $E_1 = 0.43$, $E_2 = 0.69$, $E_3 = 0.76$, $E_4 = 0.01$, $E = 1.89$.

The Fisher test ($F$-criterion) and the Student test ($t$-criterion) are used to verify the statistical significance of the results obtained.

Let us investigate the statistical significance of the constructed model, namely, make sure that all the coefficients found are valid by calculating the $F$-statistics: we propose a null hypothesis $H_0$: $R^2 = 0$ and an alternative one $H_1$: $R^2 \neq 0$. Calculate the experimental value of $F$-criterion by the formula:

$$F_{\text{exp}} = \frac{r^2}{1 - r^2} \cdot \frac{n - m - 1}{m} = 44.54$$  

(8)

and compare with $F_{\text{tabl}} = F(\alpha, k_1, k_2) = 6.42$, where $\alpha = 0.05$ – level of significance, $k_1 = m = 4$ and $k_2 = n - m - 1 = 9$ – degrees of freedom $(m$ – the number of factor variables that are included in the model, $n$ – the volume of the statistical sample). Based on the results obtained, we draw appropriate conclusions: because $F_{\text{exp}} > F_{\text{tabl}}$, then we reject the null hypothesis, but take into account the alternative hypothesis, i.e. the model as a whole is statistically significant.
When studying the regression equation, it is necessary to evaluate not only the significance of the model as a whole, but also the significance of the individual parameters $\hat{a}_0$, $\hat{a}_1$, which is provided by the statistical significance of the correlation coefficient $t$. We calculate $t$ -statistics on the formula

$$
t = \frac{R\sqrt{n-m-1}}{\sqrt{1-R^2}} = 13.35.\tag{9}
$$

Compare the result with the table value of $t$ -distribution $t_{tabl} = t(\alpha/2, k) = 2.69$, where $\alpha$ – the level of significance for the bilateral critical area, $k = n - m - 1$. Because, $|t| > t_{tabl}$ then we conclude on the reliability of the correlation coefficient, which is, essentially, the multiple correlation coefficient.

**CONCLUSIONS**

1. The finalization of this study on impact of certain macroeconomic, demographic, financial and economic factors on the revenue part of the Pension Fund of Ukraine reveals a principal clear positive linear relationship between the revenue part and the average wage. The employers’ insurance premiums rates and the number of people employed have a significant impact on the revenue part of the compulsory state social insurance fund. Taking this into account will allow to identify the major macroeconomic, demographic and financial-economic parameters, whose changes should be under special attention of the state when forecasting the income of the Pension Fund of Ukraine and consequently, strengthening the financial sources of obligatory state social insurance of Ukraine.

2. The coefficient of determination shows that the model obtained explains 95.19% of the variation of the revenue part of the budget of the Pension Fund of Ukraine. This is a sign of the high accuracy of model specification and selection of factors. The constructed model is adequate, as evidenced by an error of 5.91%, and statistically significant.

3. While maintaining the average salary, the growth rate of the revenue side of the budget of the Pension Fund of Ukraine will be 0.41, taking into account the synergistic effect of other factors considered [growth rate of single contribution ($X_3$), growth rate of employed population ($X_4$), and inflation rate ($X_5$)]. This result is rather expected and it indicates that measures should be taken to stimulate economic growth. It, in turn, should lead to an increase in the average wage.

4. While maintaining the rates of insurance premiums for employers in the funds of compulsory state social insurance of Ukraine, the growth rate of the revenue side of the budget of the Pension Fund of Ukraine will be 0.73 taking into account the synergistic effect of other factors under consideration [growth rate of average monthly wage ($X_1$), growth rate of employed population ($X_2$), and inflation rate ($X_5$)].

5. While maintaining the number of employed population, the growth rate of the revenue part of the budget of the Pension Fund of Ukraine is 0.86 taking into account the synergistic effect of other factors under consideration [average monthly wage growth rate ($X_1$), single contribution growth rate ($X_3$), and inflation rate ($X_5$)]. This result is also rather expected. It indicates that measures should be taken to stimulate economic growth, which in turn should lead to an increase in the number of employed people.

6. With all factor variables increasing by 1% [average monthly wage growth rate ($X_1$), single contribution growth rate ($X_3$), growth rate of employed population ($X_4$), and inflation rate ($X_5$)] the revenue part of the budget of the Pension Fund of Ukraine increases by 1.89%.

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Васечко Л. І., Глушак О. М., Семеняка С. О., Рамський А. Ю., Нестерова О. Д.

ДІАГНОСТИКА ДОХІДНОЇ ЧАСТИНИ ПЕНСІЙНОГО ФОНДУ УКРАЇНИ МЕТОДОМ МАТЕМАТИЧНОГО МОДЕЛЮВАНЯ

Проаналізовано проблеми вітчизняної системи державного пенсійного страхування. Зокрема, на прикладі Пенсійного фонду України (далі – ПФУ) методом математичного моделювання досліджується вплив певних макроекономічних, демографічних і фінансово-економічних чинників на формування дохідної частини бюджету ПФУ. Установлено, що визначальними параметрами, змінам яких має приділятися особлива державна увага при прогнозуванні дохідної частини Пенсійного фонду України, виступають такі ендогенні показники: динаміка середньомісячної заробітної плати і рівень інфляції (макроекономічні показники); розмір єдиного страхового внеску, що розподілений на загальнообов’язкове державне пенсійне страхування (фінансово-економічний показник); чисельність зайнятого населення (демографічний показник). Здійснено перевірку наявності мультиколінеарності між обраними факторними змінними за допомогою тесту Фаррара – Глобера. Доведено, що оскільки між факторними змінними немає високого ступеня кореляції, то етап параметризації доцільно здійснити за допомогою методу найменших квадратів (далі – МНК). На базі побудованої економетричної моделі множинної регресії було встановлено, що при збереженні середнього розміру заробітної плати темп росту дохідної частини бюджету Пенсійного фонду України буде становити 0,41 з урахуванням синергетичної дії інших факторів, що розглядаються (темпу росту величини єдиного внеску, темп ростання кількості зайнятого населення, рівня інфляції). При збереженні тарифів страхових внесків для роботодавців у фонди загальнообов’язкового державного соціального страхування України темп зростання дохідної частини бюджету Пенсійного фонду України буде становити 0,73 з урахуванням синергетичної дії інших факторів, що розглядаються (темпу росту середньомісячної заробітної плати, темп ростання кількості зайнятого населення, рівня інфляції). При збереженні кількості зайнятого населення темп росту дохідної частини бюджету Пенсійного фонду України становить 0,86 з урахуванням синергетичної дії інших факторів, що розглядаються (темпу росту середньомісячної заробітної плати, темп росту величини єдиного внеску, рівня інфляції).

Ключові слова: державне пенсійне страхування, математичні методи, економетричні моделі, кореляційно-регресійний аналіз, інформаційно-комунікаційні технології у сфері економіки

JEL Класифікація: C31, G23